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Galaxy Redshift Surveys

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Outline

1. Introduction to galaxy redshift surveys and cosmic acceleration
2. Galaxy bias
3. Dynamical probe: Redshift space distortions (RSD)
4. Geometric probe: Baryon acoustic oscillations (BAO)
5. Galaxy correlation functions in redshift space
6. Observational constraints
7. Summary

Cosmological parameters

- Density parameters

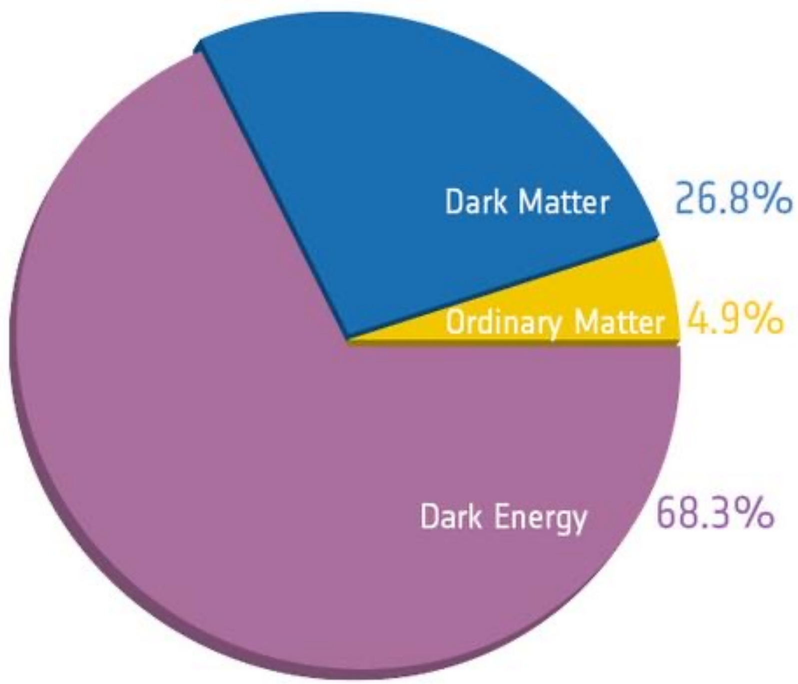
$$\Omega_i(t) = \frac{\rho_i(t)}{\rho_{crit}} = \frac{8\pi G \rho_i(t)}{3H^2(t)}$$

- Observational constraints
  - Matter: $\Omega_{m0} \sim 0.32$
  - Baryon: $\Omega_{b0} \sim 0.05$
  - Dark matter: $\Omega_{dm0} \sim 0.27$
  - Radiation: $\Omega_{r0} \sim 10^{-3}$
  - Curvature: $\Omega_{K0} \sim 0$
  - Dark energy: $\Omega_{\Lambda 0} \sim 0.68$
  - Hubble constant: $H_0 \sim 68$

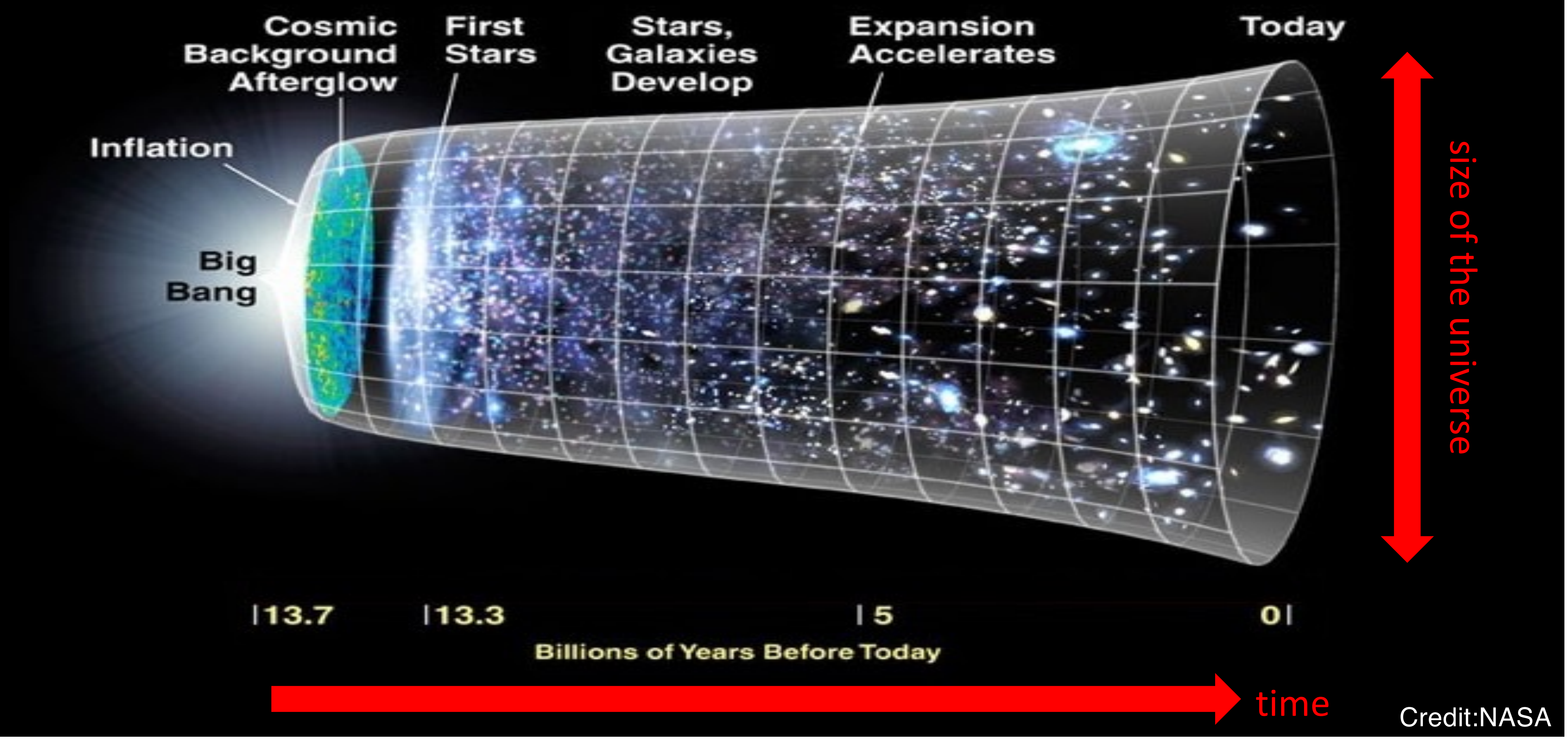
$\rightarrow \Omega_{m0} + \Omega_{\Lambda 0} = 1 + \Omega_{K0} \sim 1$

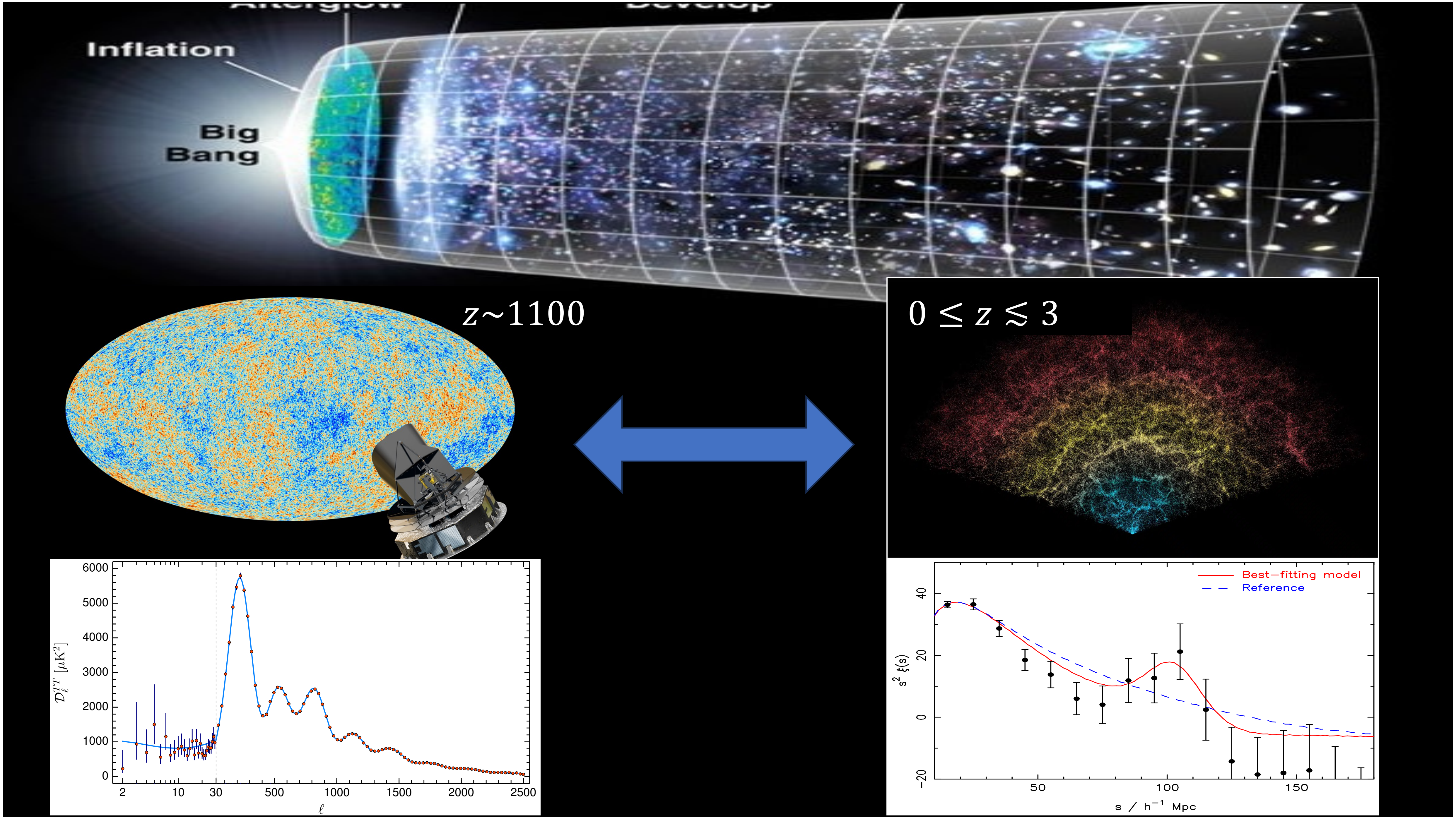
- Friedmann equations

$$H^2(a) = H_0^2 \left(\frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0} - \frac{\Omega_{K0}}{a^2} \right)$$



Expansion history of our Universe





Inflation
Big Bang
$z\sim1100$
$0 \le z \lesssim 3$
$\mathcal{D}_\ell^{TT}$ [μK^2]
ℓ
Best−fitting model
Reference
$s^2\,\xi(s)$
s / h^{-1} Mpc

Cosmic acceleration in the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

Geometry of space time = Matter distribution

- Matter contents of the universe determine its gravitational field.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\left(T_{\mu\nu} - \frac{\Lambda}{8\pi G} g_{\mu\nu}\right)$$

← Add new matter "Dark energy"

or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

← Modify geometry "Modified gravity"

Dark energy candidates

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

- Cosmological constant Λ
  - $w(a) = -1$
- Constant but not Λ
  - $w(a) = w_0 \neq -1$
- Dynamical dark energy
  - $w(a) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$
- Modification of gravitational law
  - We are allowed to keep the term in the LHS.

Equation of state w

$$w = p/\rho$$

- radiation: w=1/3
- matter: w=0
- cosmological constant: w=-1

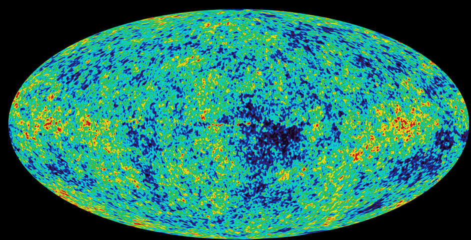
Cosmological constant problem

Theory: $\Omega_{\mathrm{vac0}} \sim 3\times10^{123}$

↕

Observation: $\Omega_{\Lambda 0} \sim 0.68$

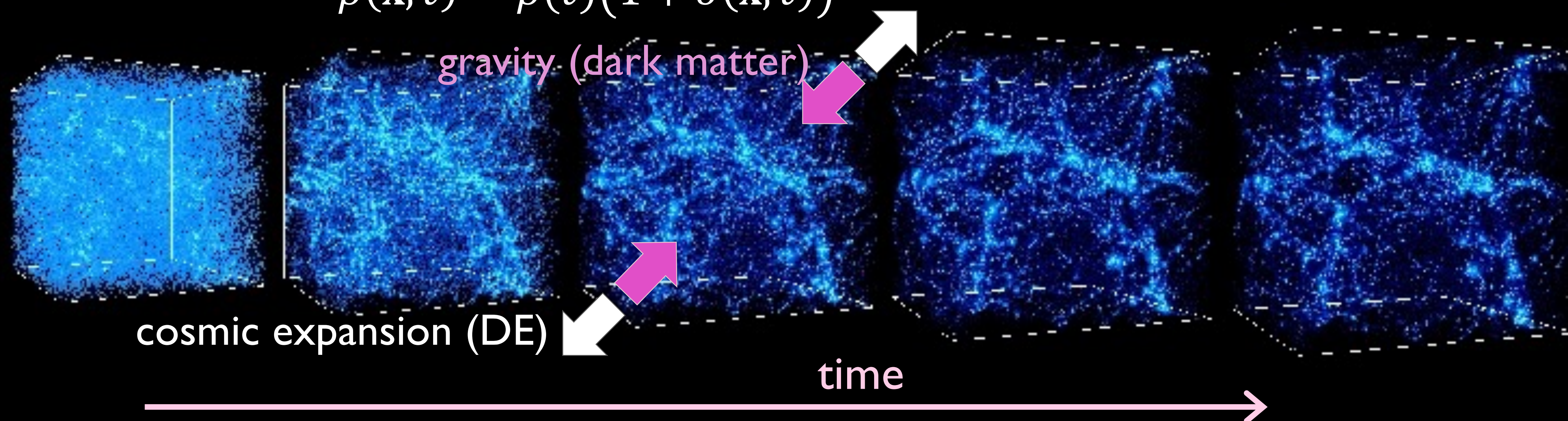
Structure grows through gravity in the expanding universe



$T(\theta) = \bar{T}\big(1 + \Delta T(\theta)\big)$

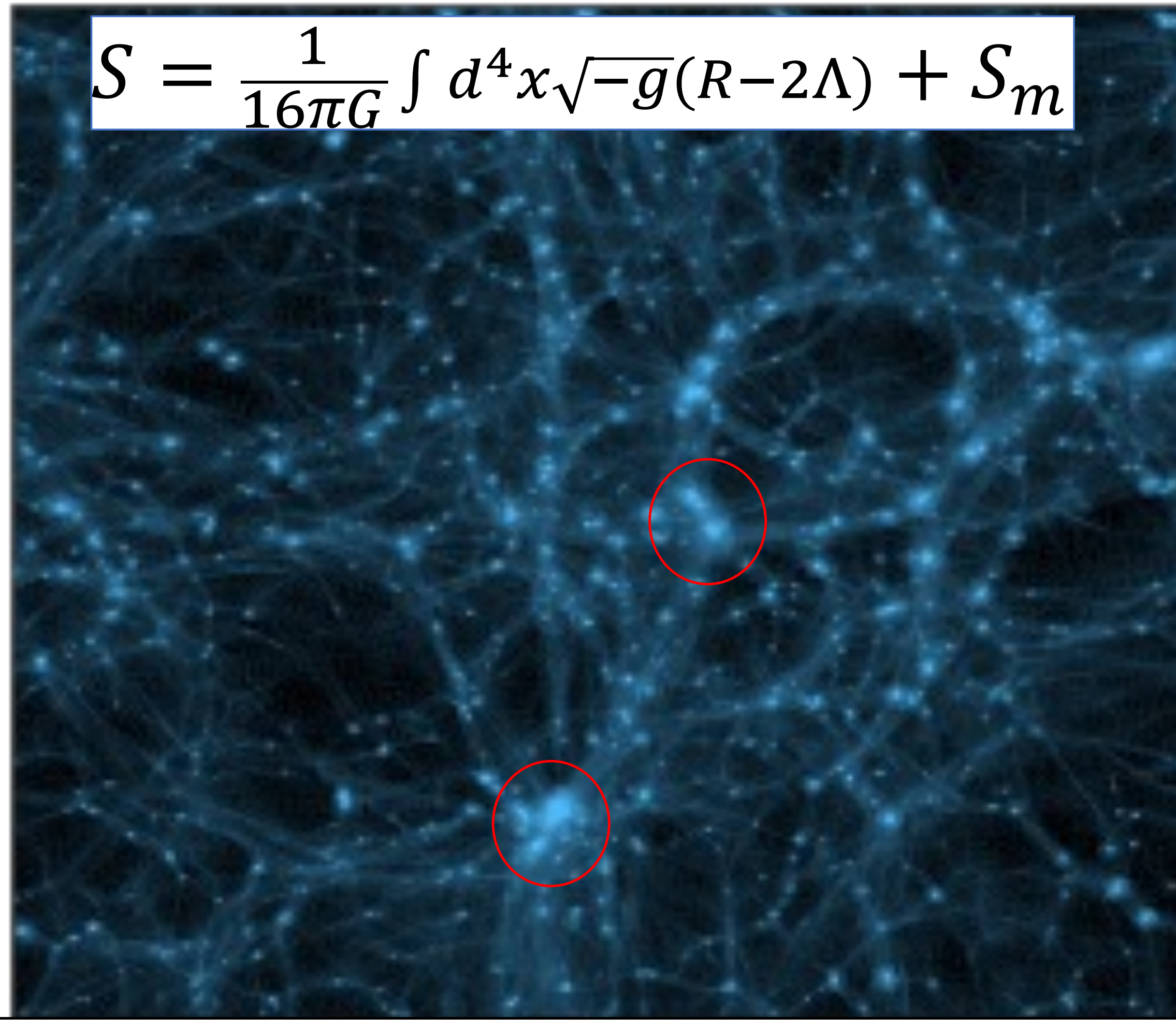
CMB -> initial conditions

$\rho(\mathbf{x}, t) = \bar{\rho}(t)\big(1 + \delta(\mathbf{x}, t)\big)$



N-body simulation of matter distribution under general relativity

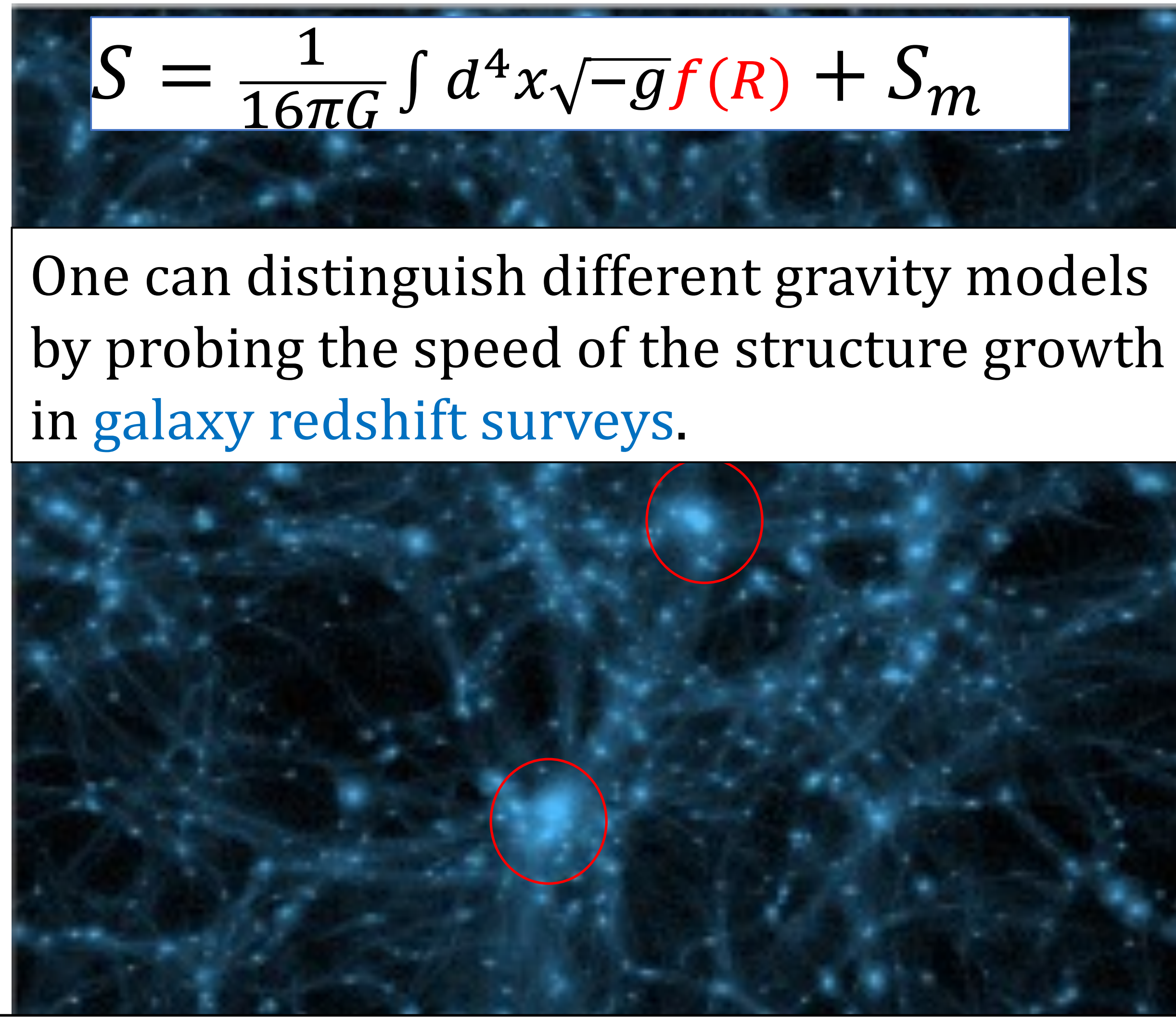
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R-2\Lambda) + S_m$$



Figures taken from G.B. Zhao et al

N-body simulation of matter distribution under one modified gravity model

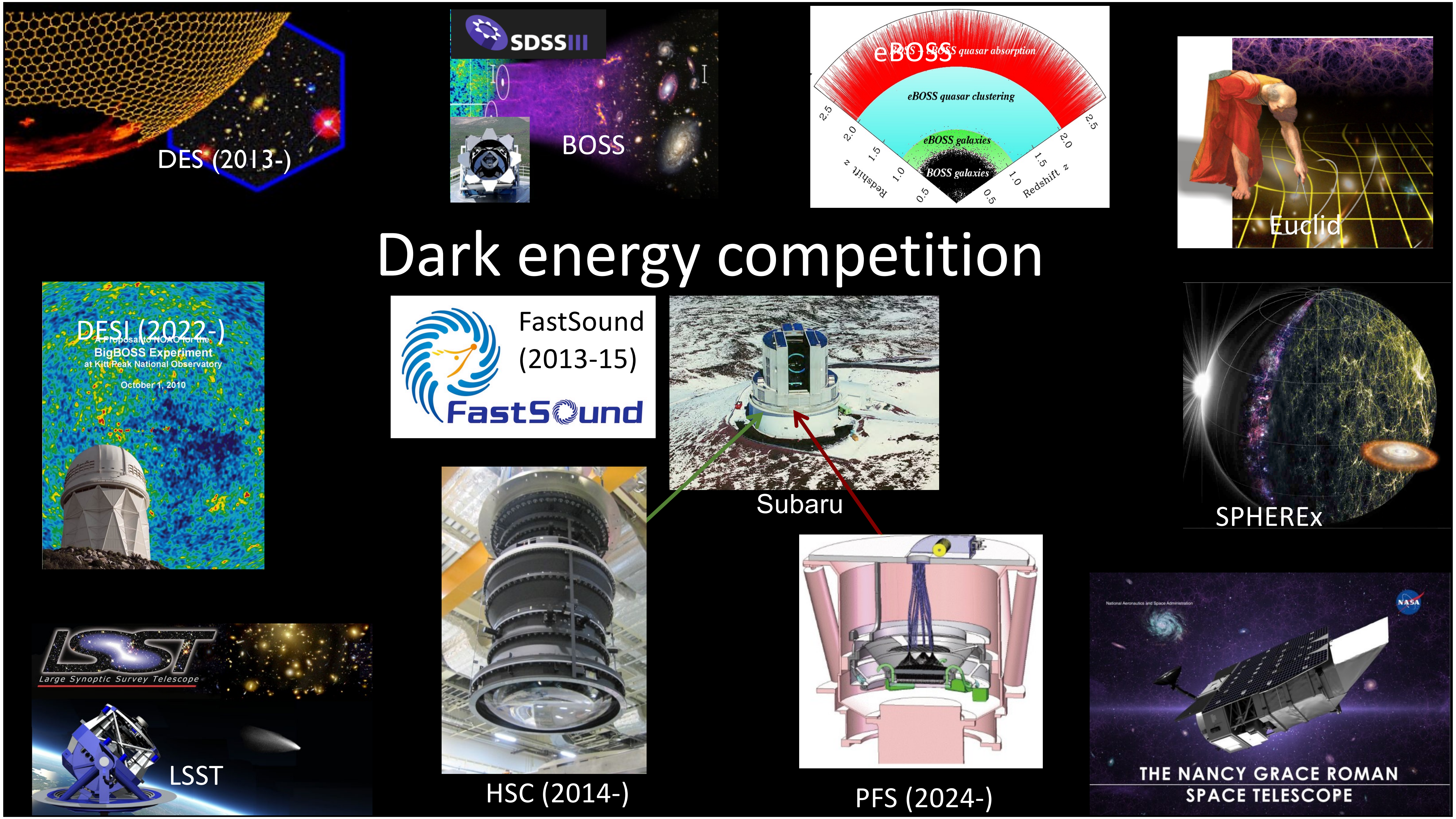
- Same initial condition, but *f*(*R*) gravity model assumed.



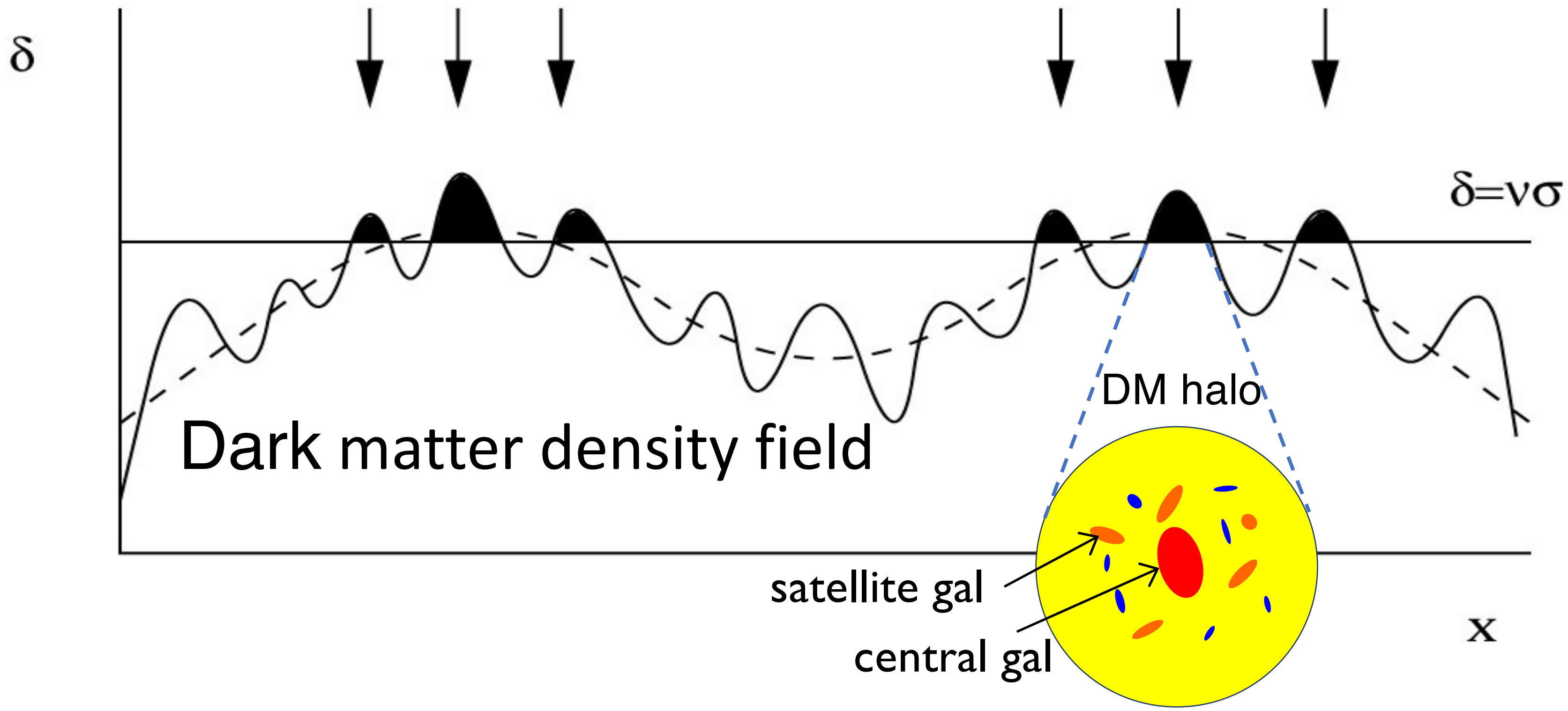
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + S_m$$

One can distinguish different gravity models by probing the speed of the structure growth in galaxy redshift surveys.

Figures taken from G.B. Zhao et al


Dark energy competition
DES (2013-)
SDSSIII
BOSS
eBOSS
eBOSS quasar clustering
eBOSS galaxies
BOSS galaxies
Redshift z
Redshift z
Euclid
DESI (2022-)
A Proposal to NOAO for the
BigBOSS Experiment
at Kitt Peak National Observatory
October 1, 2010
FastSound
(2013-15)
FastSound
Subaru
SPHEREx
HSC (2014-)
PFS (2024-)
Large Synoptic Survey Telescope
LSST
NASA
THE NANCY GRACE ROMAN
SPACE TELESCOPE

Bias:
From dark matter to dark matter halos and galaxies:



- Mass density ≠ galaxy number density

$$\delta_m(\mathbf{x},t) = \frac{\rho_m(\mathbf{x},t)}{\bar{\rho}_m(t)} - 1$$

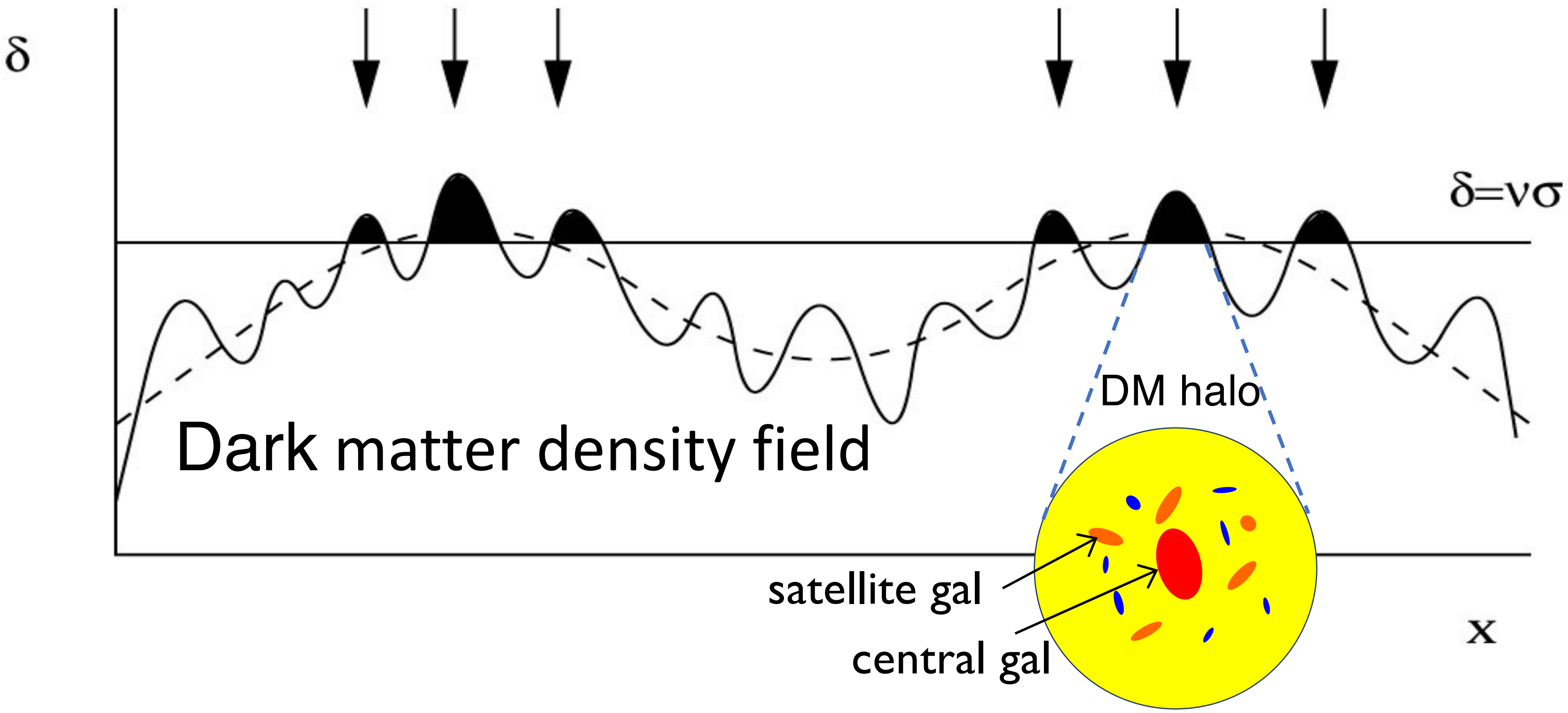
$$\updownarrow$$

$$\delta_g(\mathbf{x},t) = \frac{\rho_g(\mathbf{x},t)}{\bar{\rho}_g(t)} - 1$$

- Galaxy distribution is a biased tracer of underlying matter distribution (Kaiser 1984, Bardeen+1986)

$$\delta_g(\mathbf{x},t) = F[\delta_m(\mathbf{x}',t)]$$

Bias:
From dark matter to dark matter halos and galaxies:



$$\delta_g(\mathbf{x}, t) = F[\delta_m(\mathbf{x}', t)]$$

- The relation between galaxy and dark matter distribution is extremely complicated.
- The simplest bias model: "linear bias"

$$\delta_g(\mathbf{x}) = b\delta_m(\mathbf{x})$$

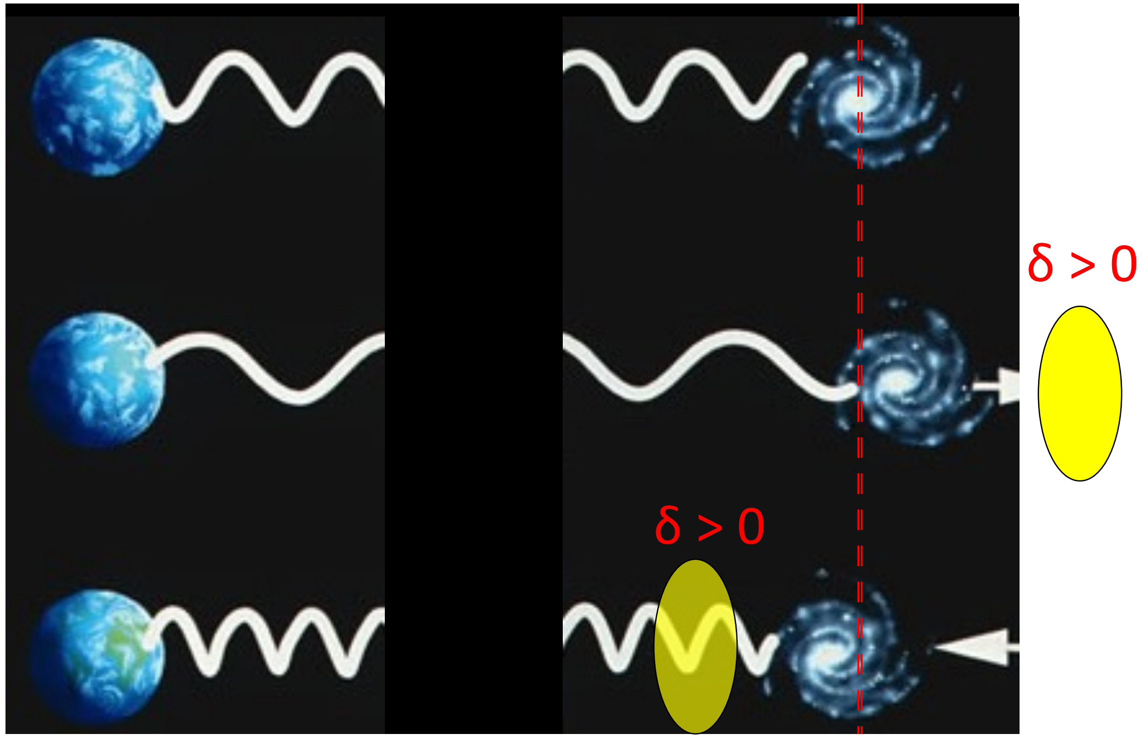
- One can consider higher-order bias
- Effective field theory

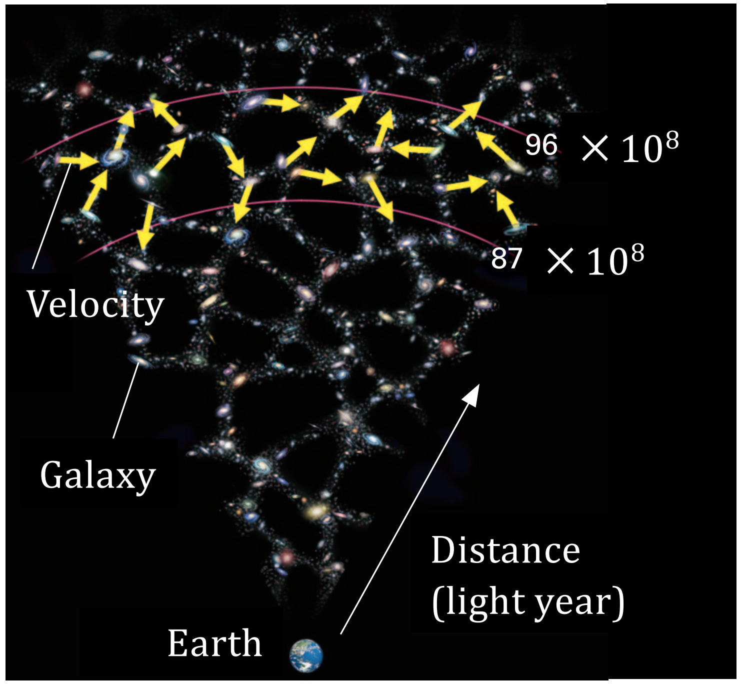
Redshift space distortions (RSD)

- Radial coordinates of galaxies are measured through redshift z (Doppler shift)

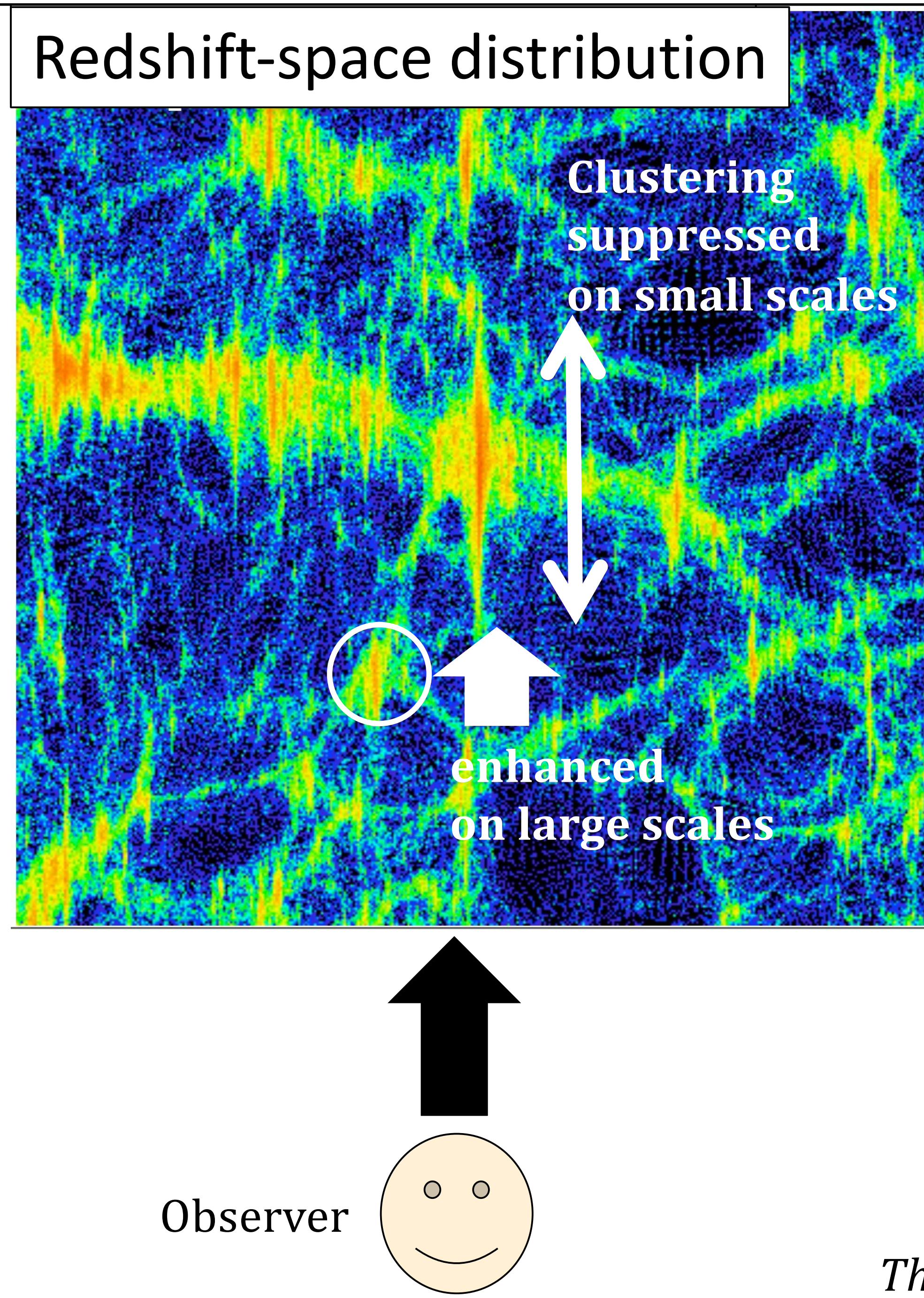
Redshift cz = radial peculiar velocity + $aH(a)r$

expansion





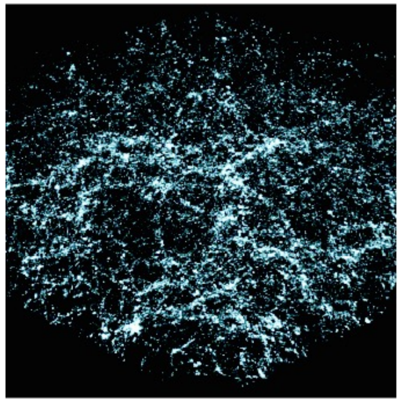
- Only radial positions are affected by RSD, not angular positions.

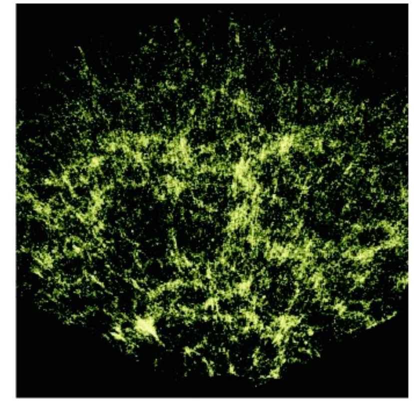


We cannot measure true distances to galaxies. Will it be a serious problem?

The visualization made by T. Nishimich

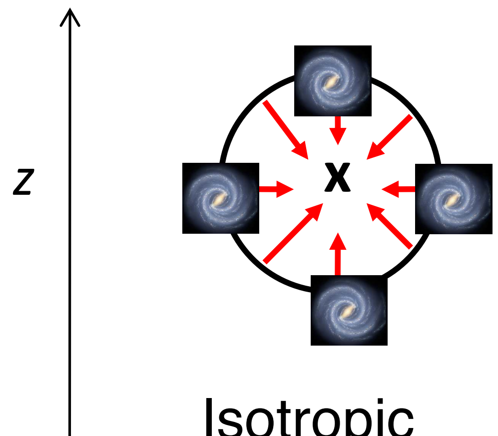
No, RSD tells velocity field (= speed of growth)



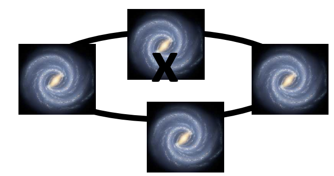


redshift $cz = aH(a)r + v_p$

Real-space to redshift –space mapping



$$\nabla \cdot v = -aHf\delta(x)$$



$$v \sim d\sigma_8/d \ln a = f\sigma_8$$

Isotropic

Squashed along LoS

$$f = d \ln\delta/d \ln a = \Omega_m{}^{\gamma}$$

γ=0.556 (GR)

γ=0.683(DGP gravity)

RSD = coordinate transformation

- Number conservation $n_s(\vec{x}_s)d^3x_s = n(\vec{x})d^3x$

  Redshift-space Real-space

- Coordinate transformation

$$x_s = x + \frac{\vec{v}(\vec{x})\cdot\hat{x}}{H_0} \qquad \rho(\vec{x}) = \bar{\rho}[1+\delta(\vec{x})], \quad \delta << 1$$

- Jacobian

$$J = \left|\frac{d^3x}{d^3x_s}\right| \simeq 1 - \frac{\partial}{\partial x}\left[\frac{\vec{v}\cdot\hat{x}}{H_0}\right]$$

- Density field

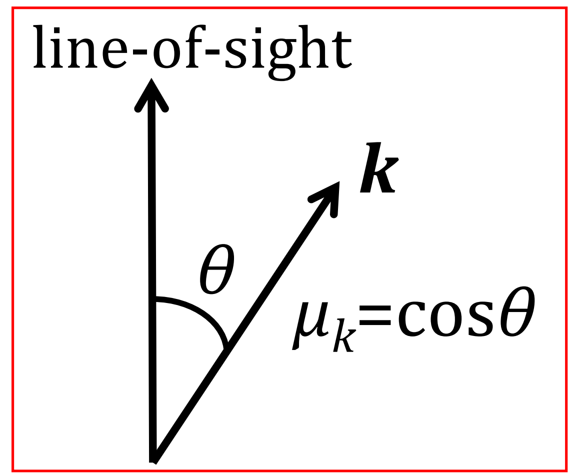
$$\delta_s(\vec{x}) = \delta(\vec{x}) - \frac{\partial}{\partial x}\left[\frac{\vec{v}\cdot\hat{x}}{H_0}\right]$$

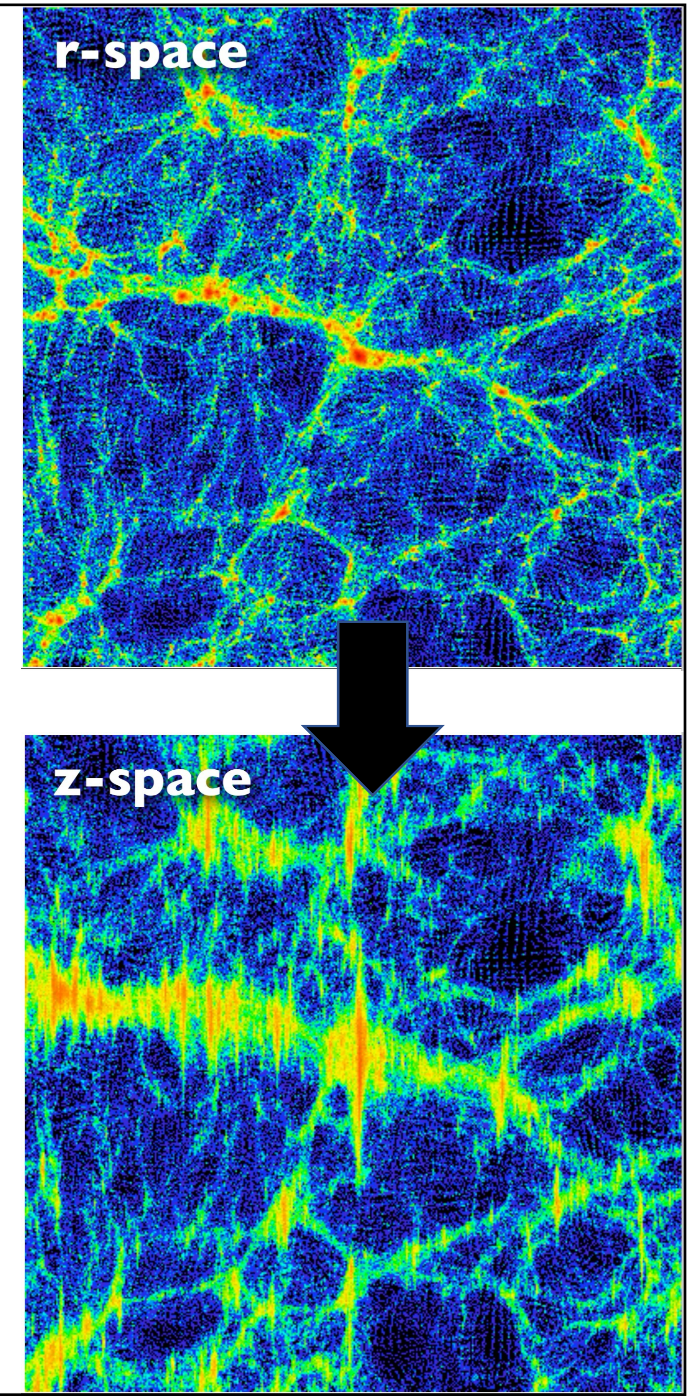
- Fourier-space density

$$\delta_s(\vec{k}) = \delta(\vec{k}) + \int \frac{d^3k'}{(2\pi)^3}\delta(\vec{k}')[f(\widehat{k}'\cdot\hat{z})^2]\int d^3x e^{i(\vec{k'}-\vec{k'})\cdot\vec{x}}$$

$$= [1 + f\mu_k^2]\delta(\vec{k})$$

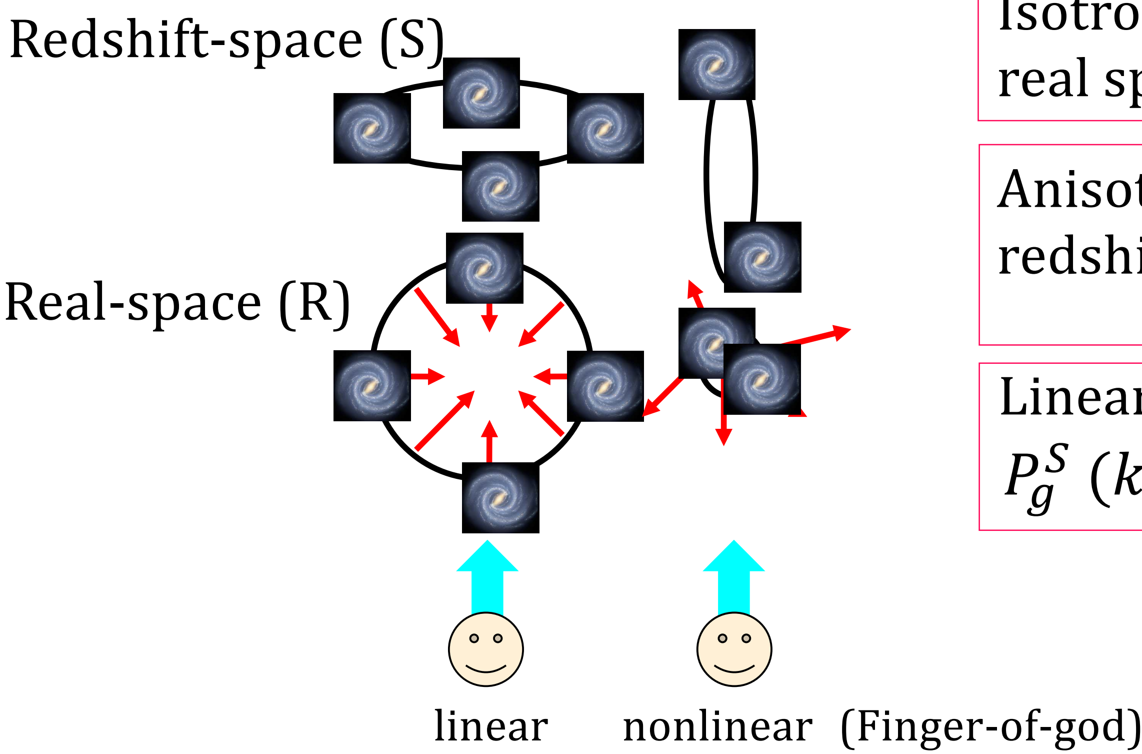
$\mu_{\mathbf{k}}$ is defined to be $\hat{z}\cdot\hat{k}$



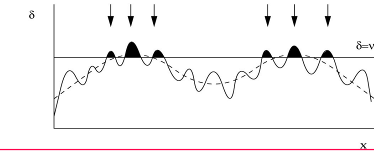


Redshift-space power spectrum

$$\rho(\vec{x}) = \bar{\rho}[1+\delta(\vec{x})], \quad \delta << 1$$



Isotropic in real space $\quad \delta_g(k) = b\delta_m(k)$



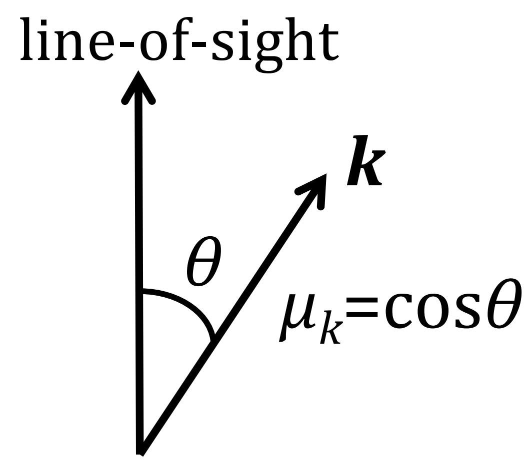
Anisotropy in redshift space

$$\delta_g^S(k,\mu_k) = (b + f\mu_k^2)\delta_m(k)$$

$$f(a) = d\ln D/d\ln a = \Omega_m^{0.55}$$

Linear power spectrum (Kaiser 1987)

$$P_g^S(k,\mu_k) = (b + f\mu_k^2)^2 P_m(k) \times \exp(-k^2\mu_k^2\sigma_v^2)$$



Redshift-space power spectrum

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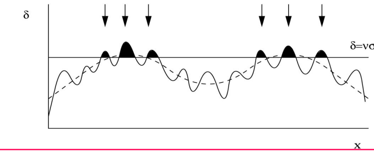
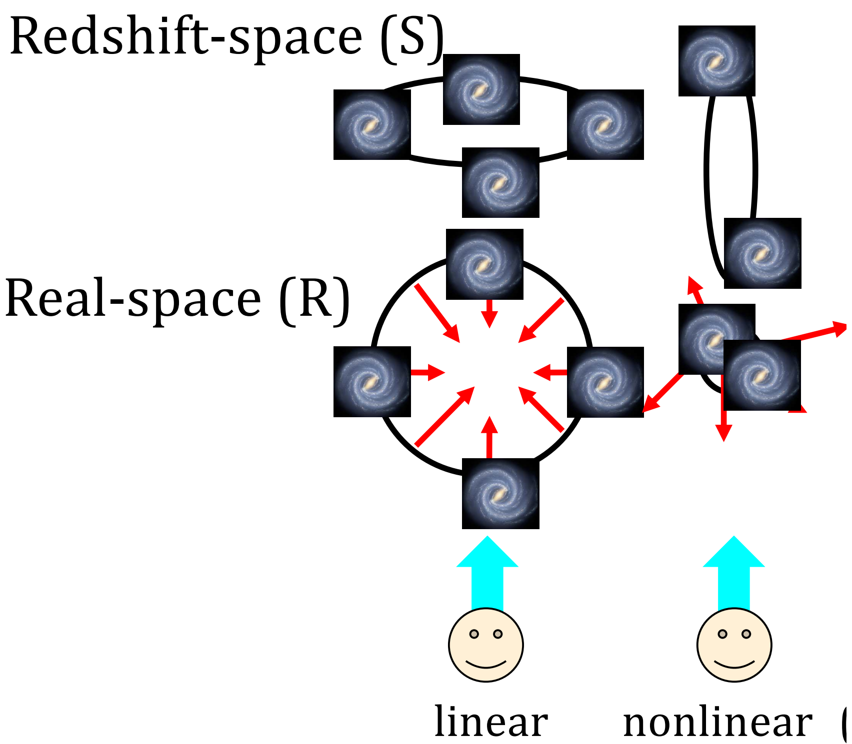




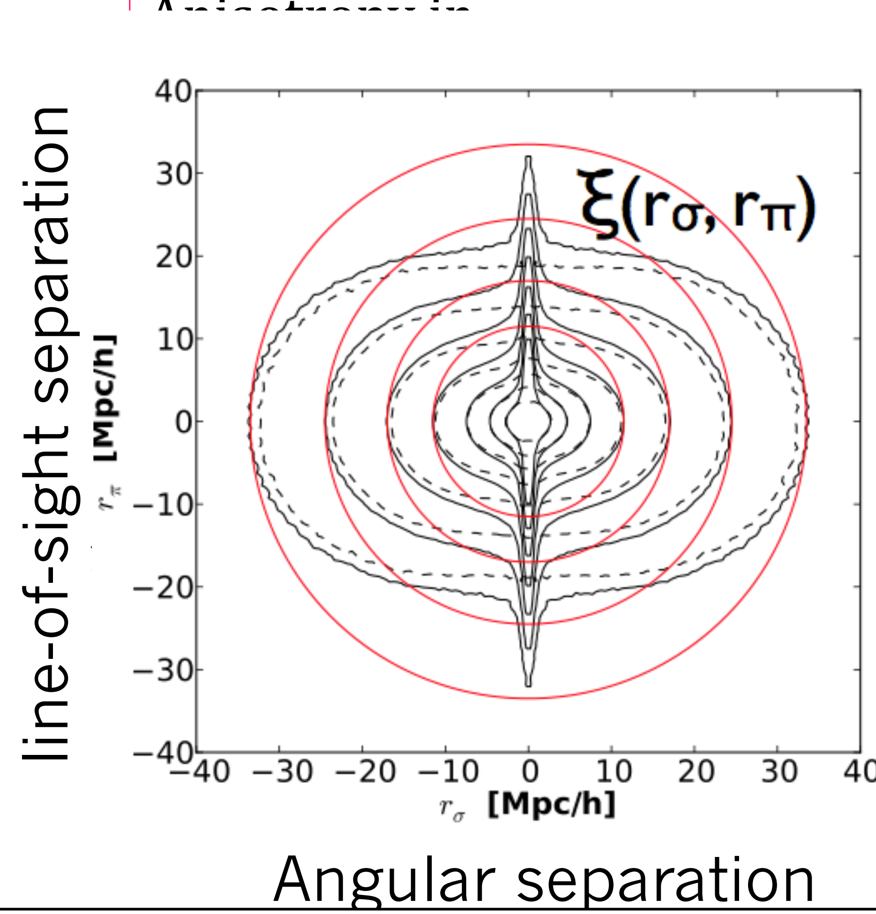
Anisotropy in

$+ f\mu_k^2)\delta_m(k)$

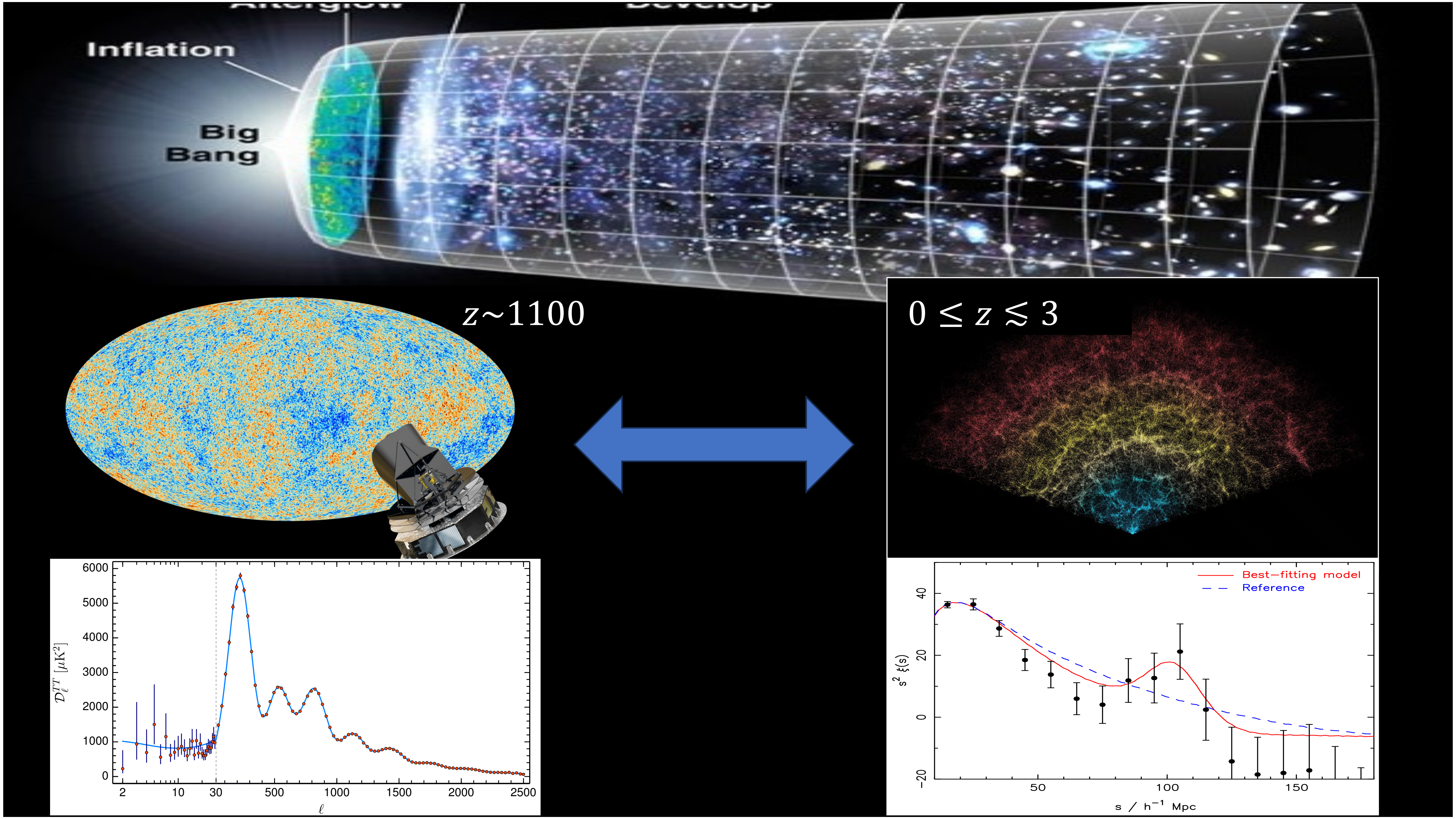
$l \ln a = \Omega_m^{0.55}$

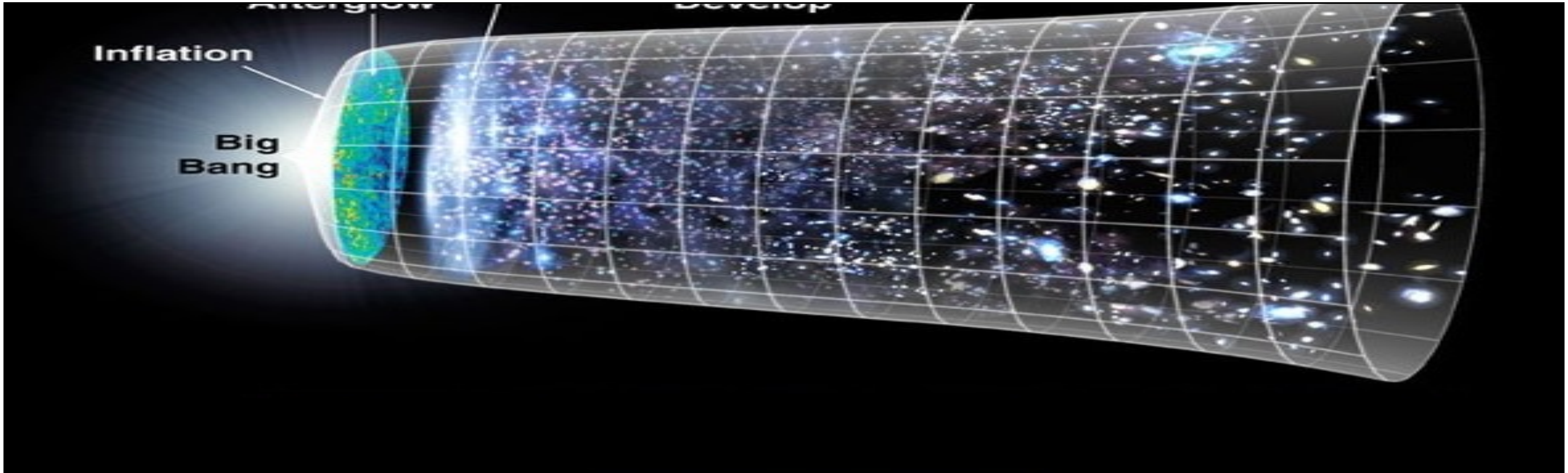
987)

$)\times\exp(-k^2\mu_k^2\sigma_v^2)$

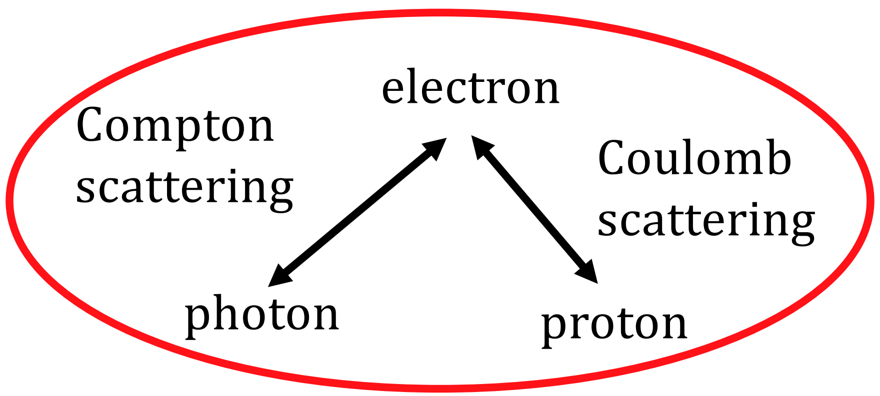


White+(2011)

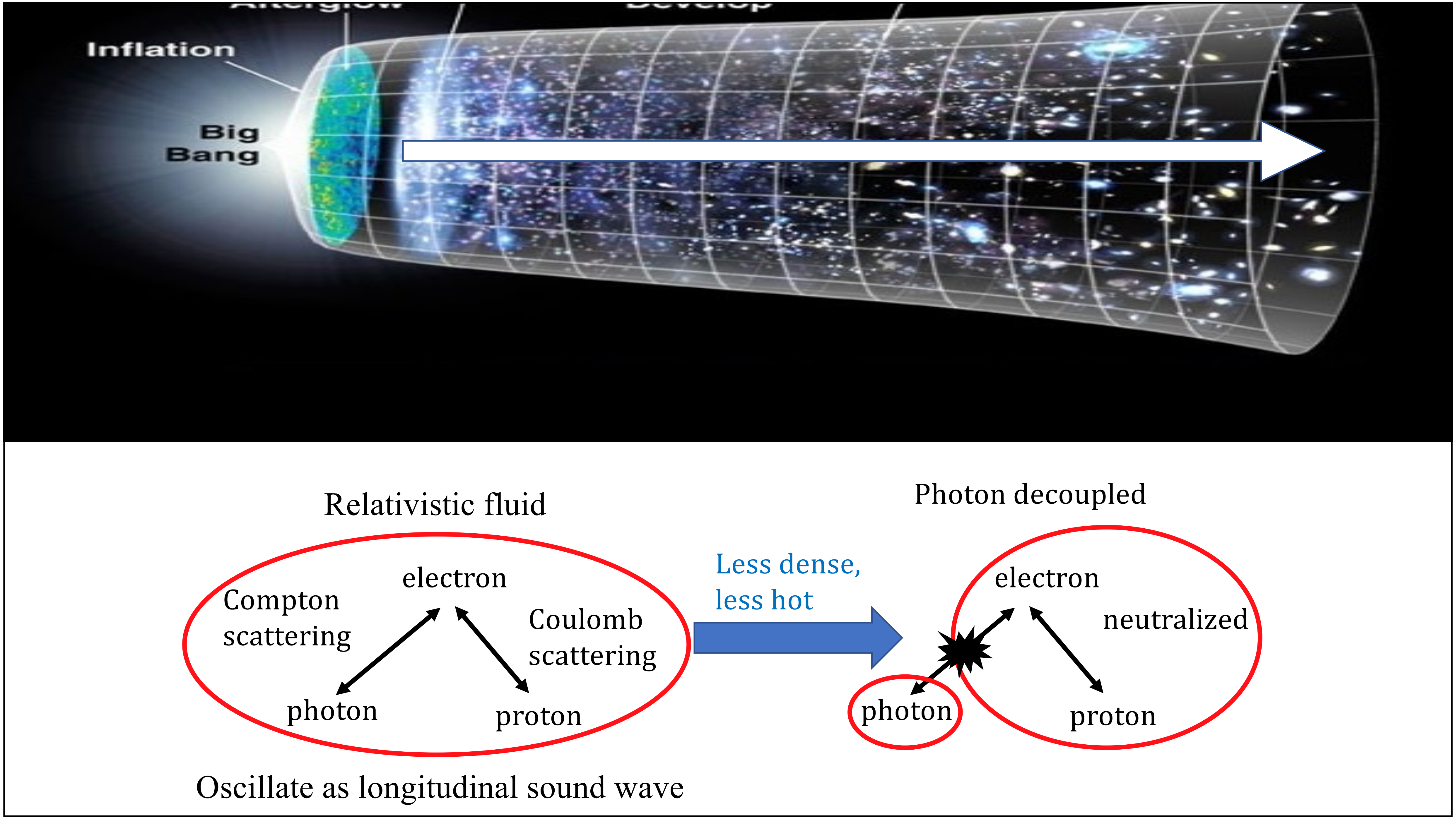

Inflation
Big Bang
z~1100
0 ≤ z ≲ 3
6000
5000
4000
3000
2000
1000
0
2
10
30
500
1000
1500
2000
2500
ℓ
Best−fitting model
Reference
40
20
0
−20
50
100
150
s / h⁻¹ Mpc



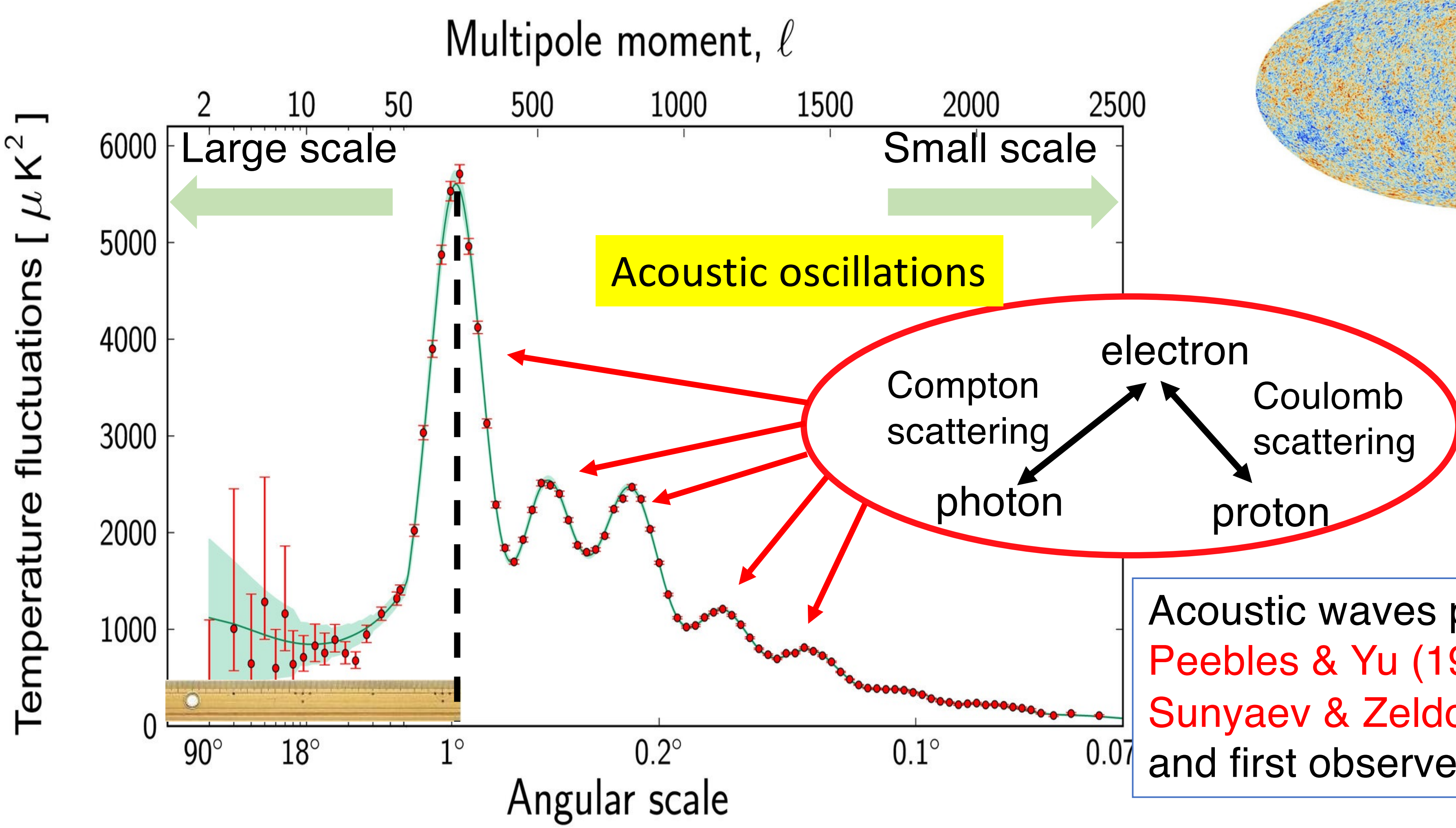
Relativistic fluid

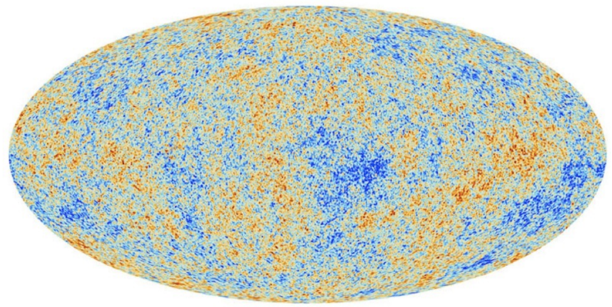


Oscillate as longitudinal sound wave


Inflation
Big
Bang
Relativistic fluid
electron
Compton
scattering
Coulomb
scattering
photon
proton
Oscillate as longitudinal sound wave
Less dense,
less hot
Photon decoupled
electron
neutralized
photon
proton

Power spectrum of cosmic microwave background

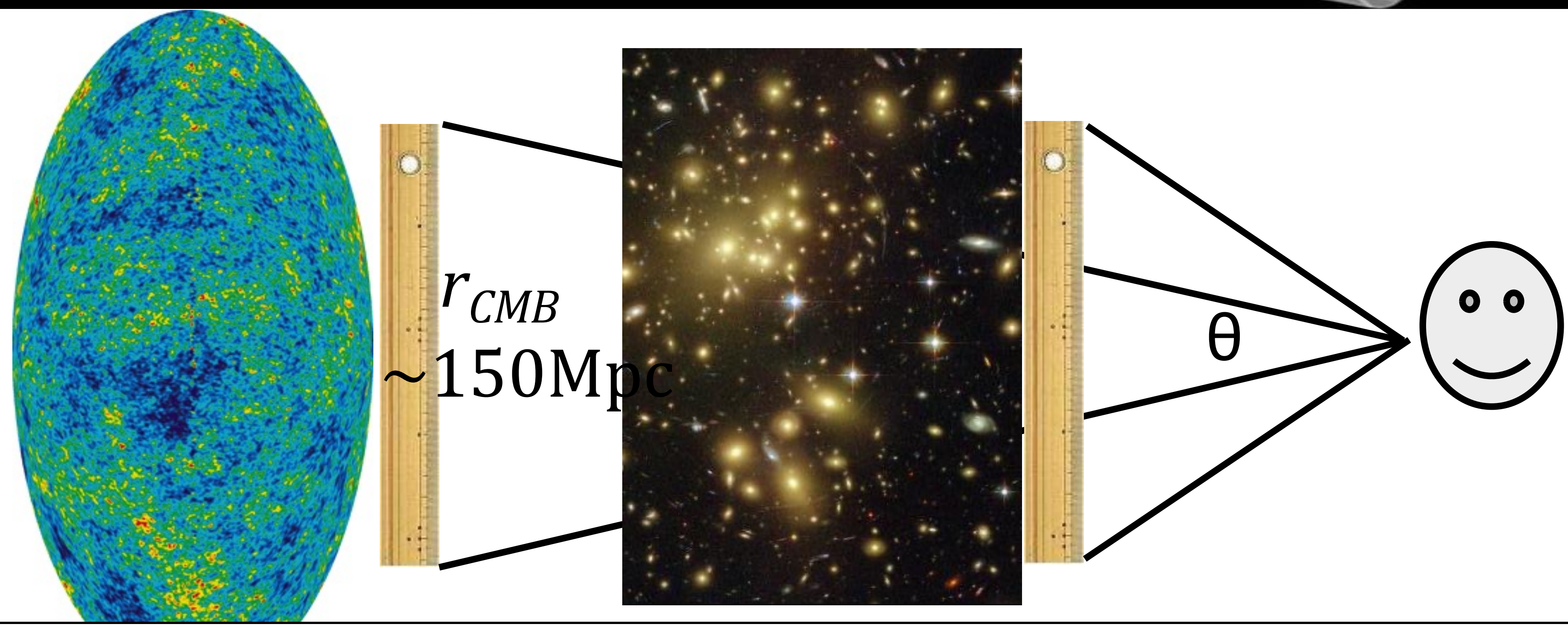




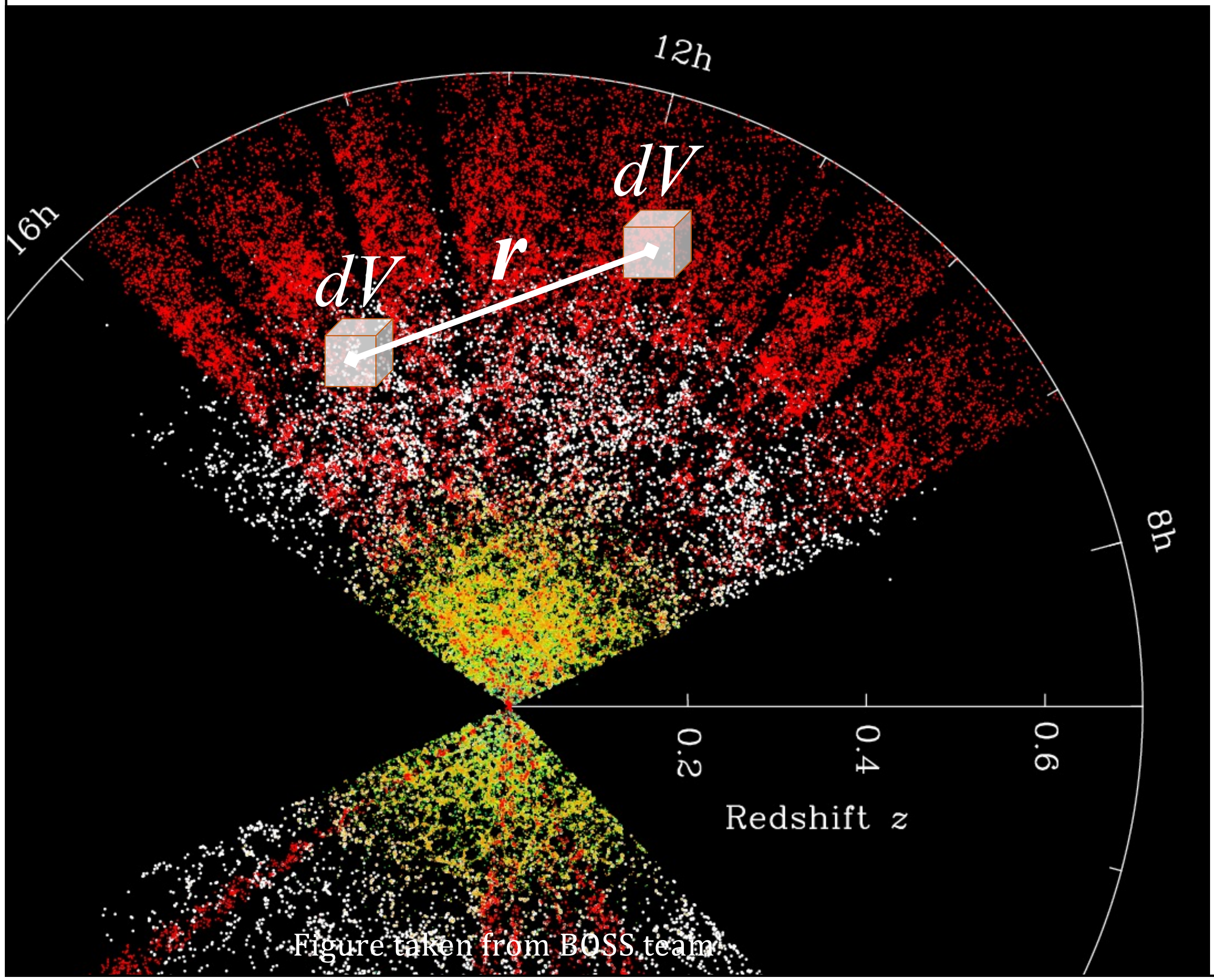
Acoustic waves predicted by Peebles & Yu (1970) and Sunyaev & Zeldovich (1970) and first observed in 2000

Planck collaboration+

Acoustic oscillation features are also imprinted to large-scale structure



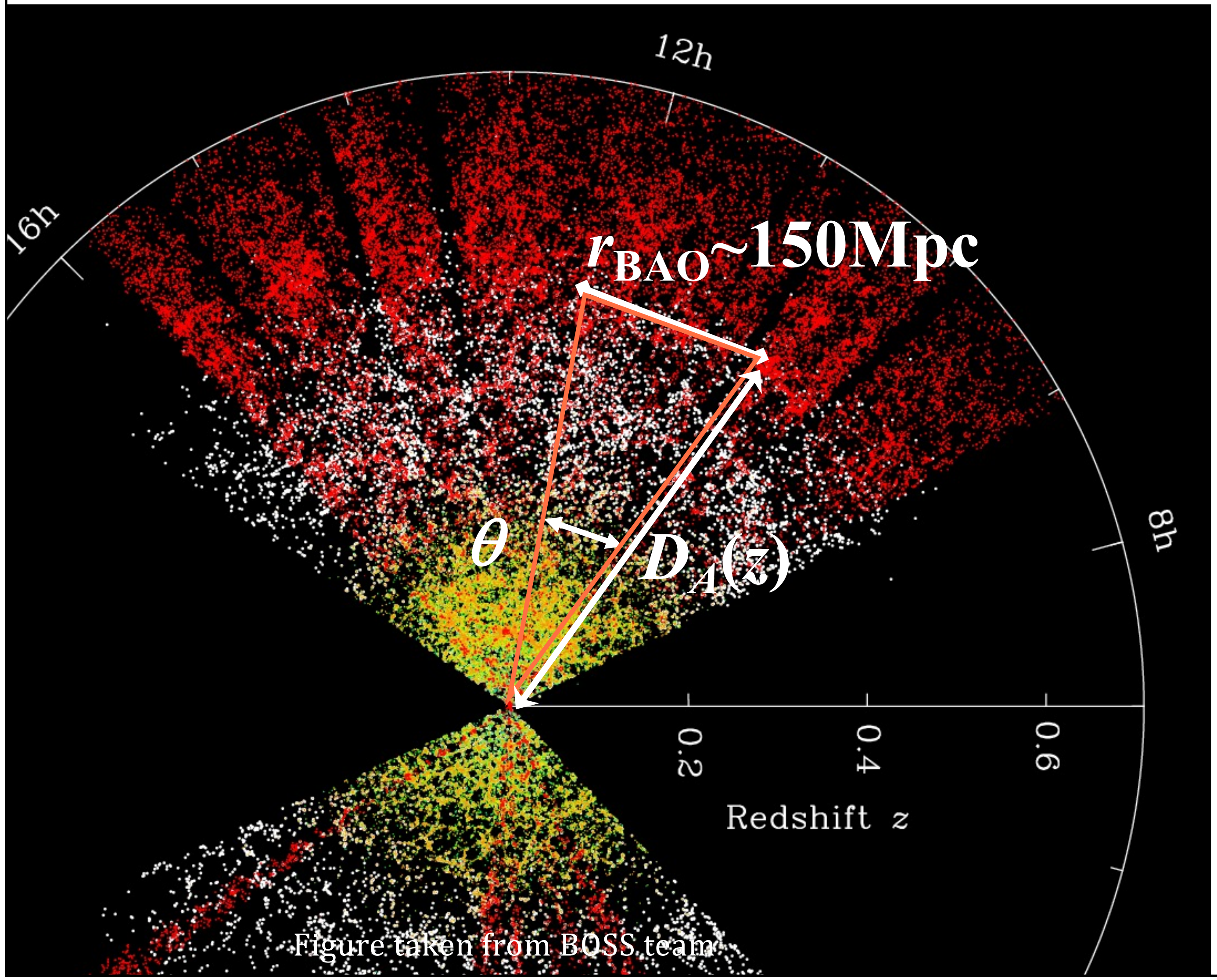
Measuring galaxy distribution and acoustic features



- Measure galaxy clustering strengths: two-point correlation function

$$dP = \bar{n}_g^2 \left[1 + \xi_g(r) \right] dV^2$$

Measuring galaxy distribution and acoustic features



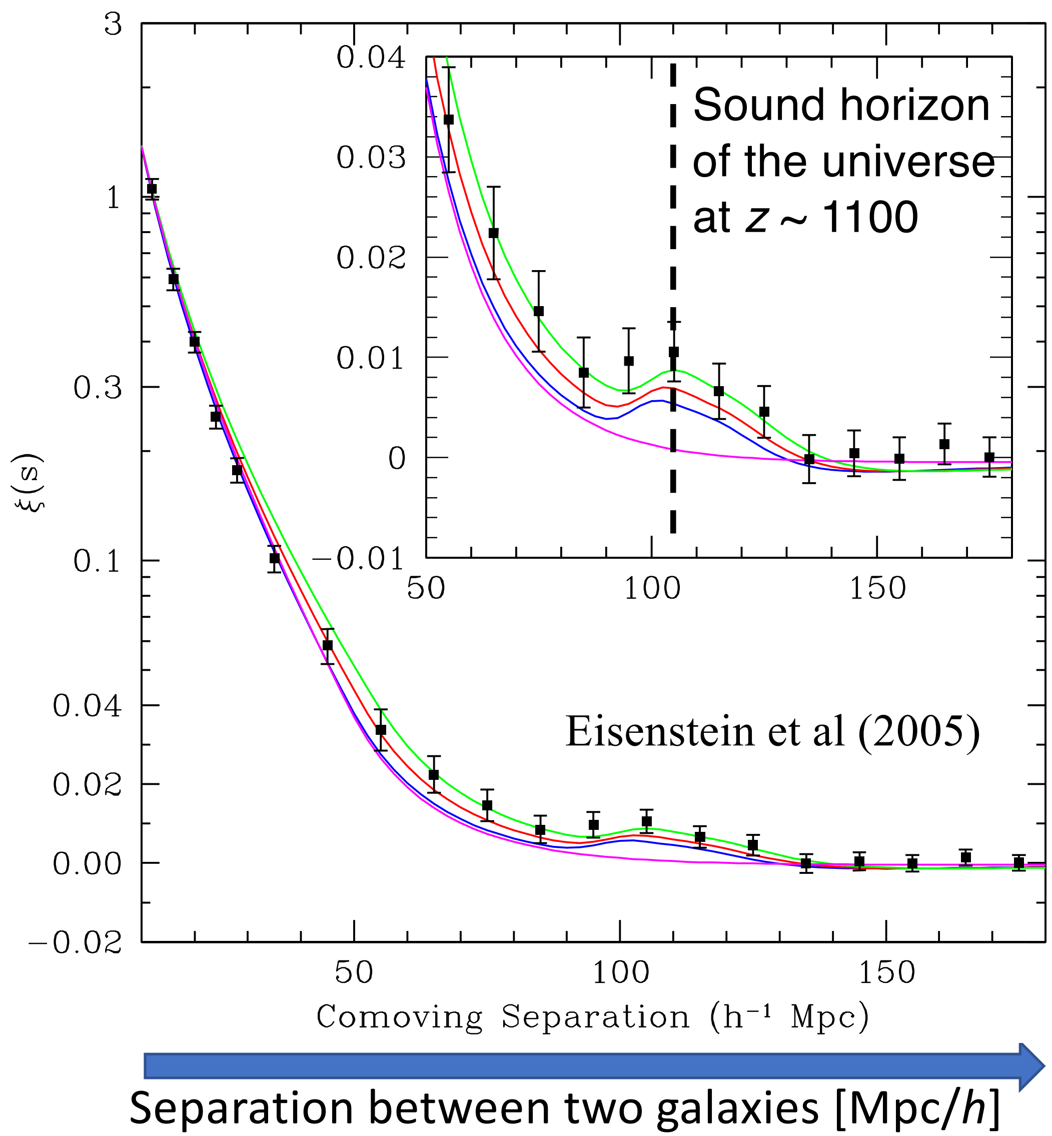
- Measure galaxy clustering strengths: two-point correlation function

$$dP = \bar{n}_g^2\left[1+\xi_g(r)\right]dV^2$$

- Find a tiny excess in the galaxy pairs at BAO scale (a priori known from CMB to be ~150Mpc)

$$r_{\rm BAO} = D_A(z)\theta_{\rm obs}$$

Measuring galaxy distribution and acoustic features



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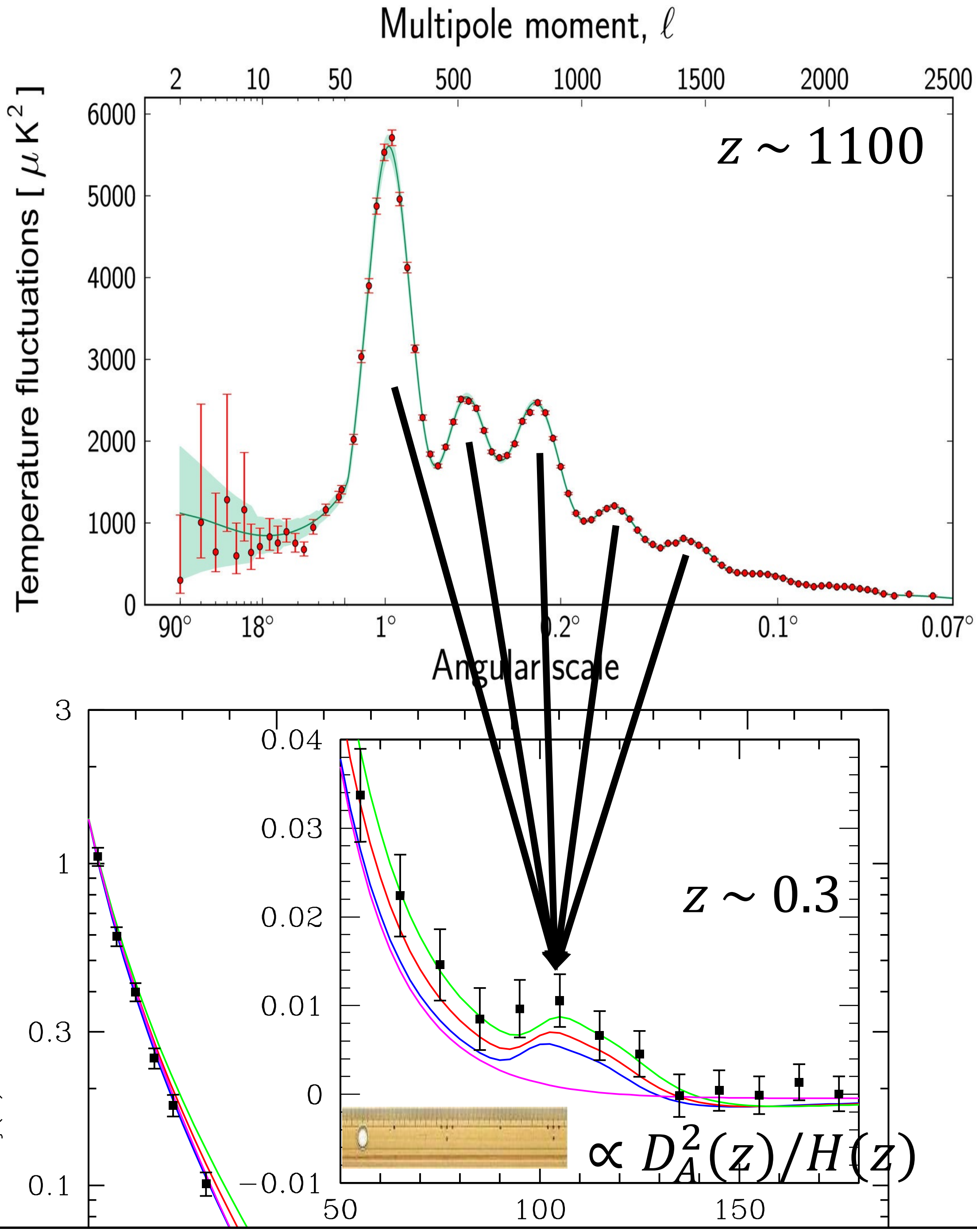
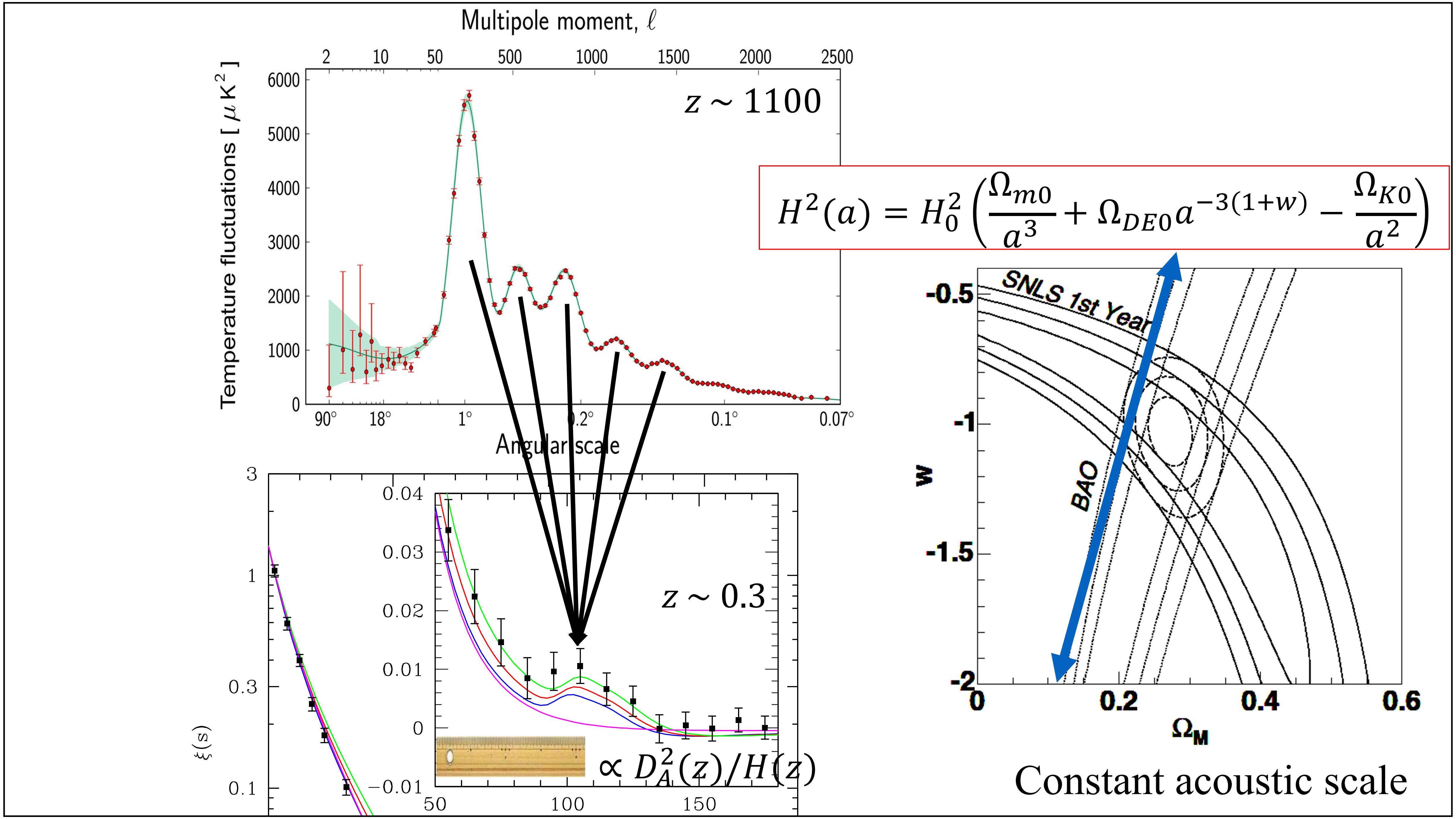
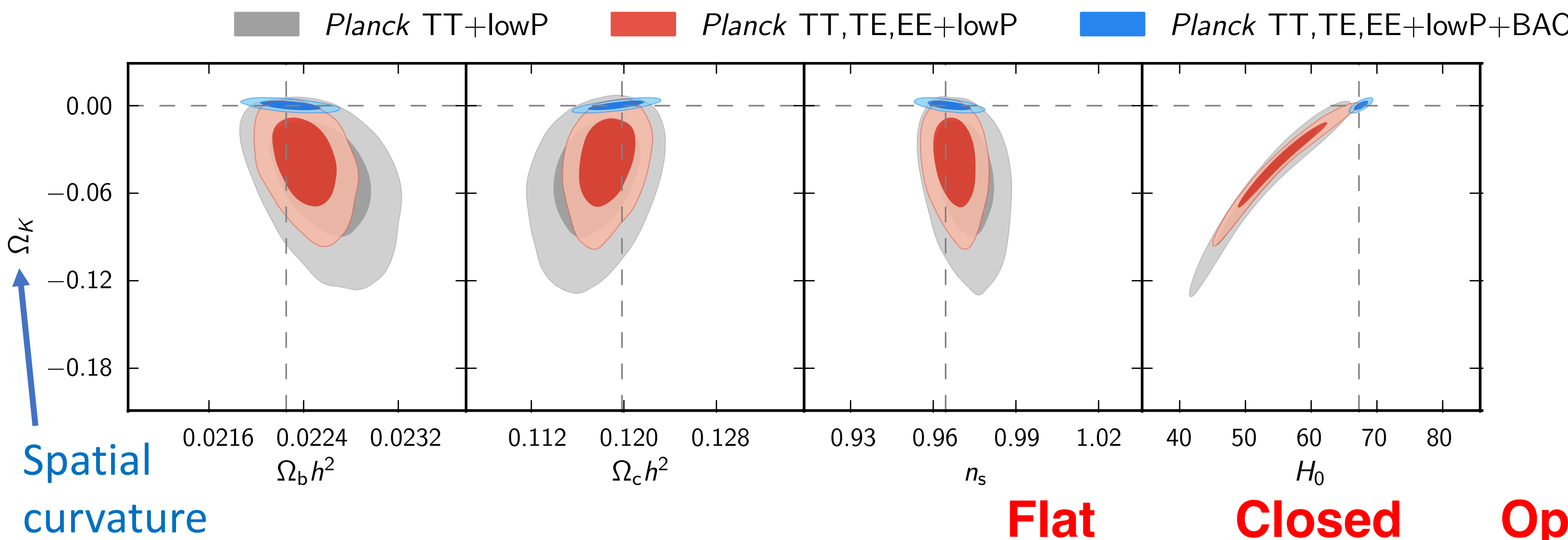


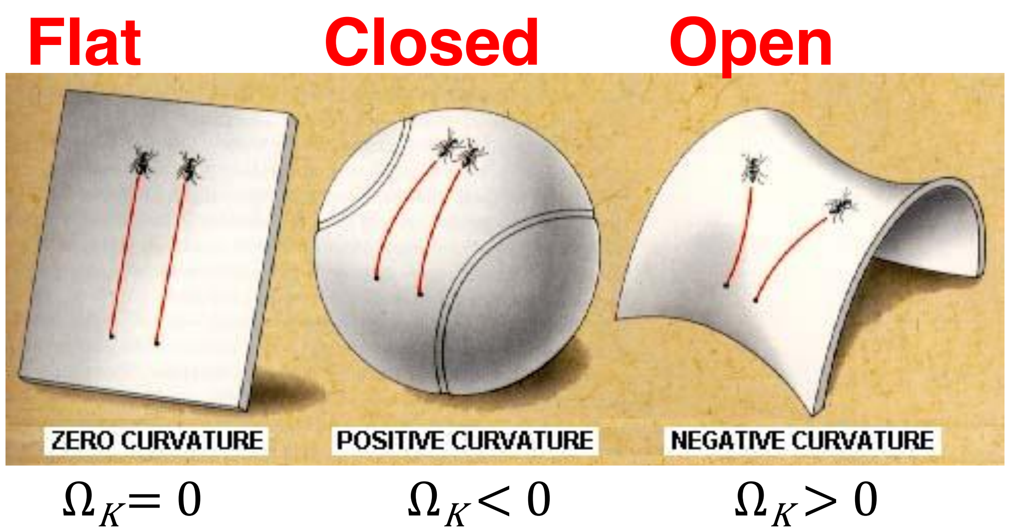
- Acoustic features imprinted at z ~ 1100 remain in the later universe, z < 1.
- The feature provides the size of the universe (sound horizon) when its age was 380,000 years after the bigbang.
- This feature in galaxy distribution enables us to constrain the geometry of the universe, hence H_0, dark energy, spatial curvature etc.



Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK²]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale
$z \sim 1100$
$$H^2(a) = H_0^2\left(\frac{\Omega_{m0}}{a^3} + \Omega_{DE0}a^{-3(1+w)} - \frac{\Omega_{K0}}{a^2}\right)$$
SNLS 1st Year
BAO
-0.5
-1
-1.5
-2
w
0
0.2
0.4
0.6
Ω_M
3
1
0.3
0.1
ξ(s)
0.04
0.03
0.02
0.01
0
−0.01
50
100
150
$z \sim 0.3$
$\propto D_A^2(z)/H(z)$
Constant acoustic scale

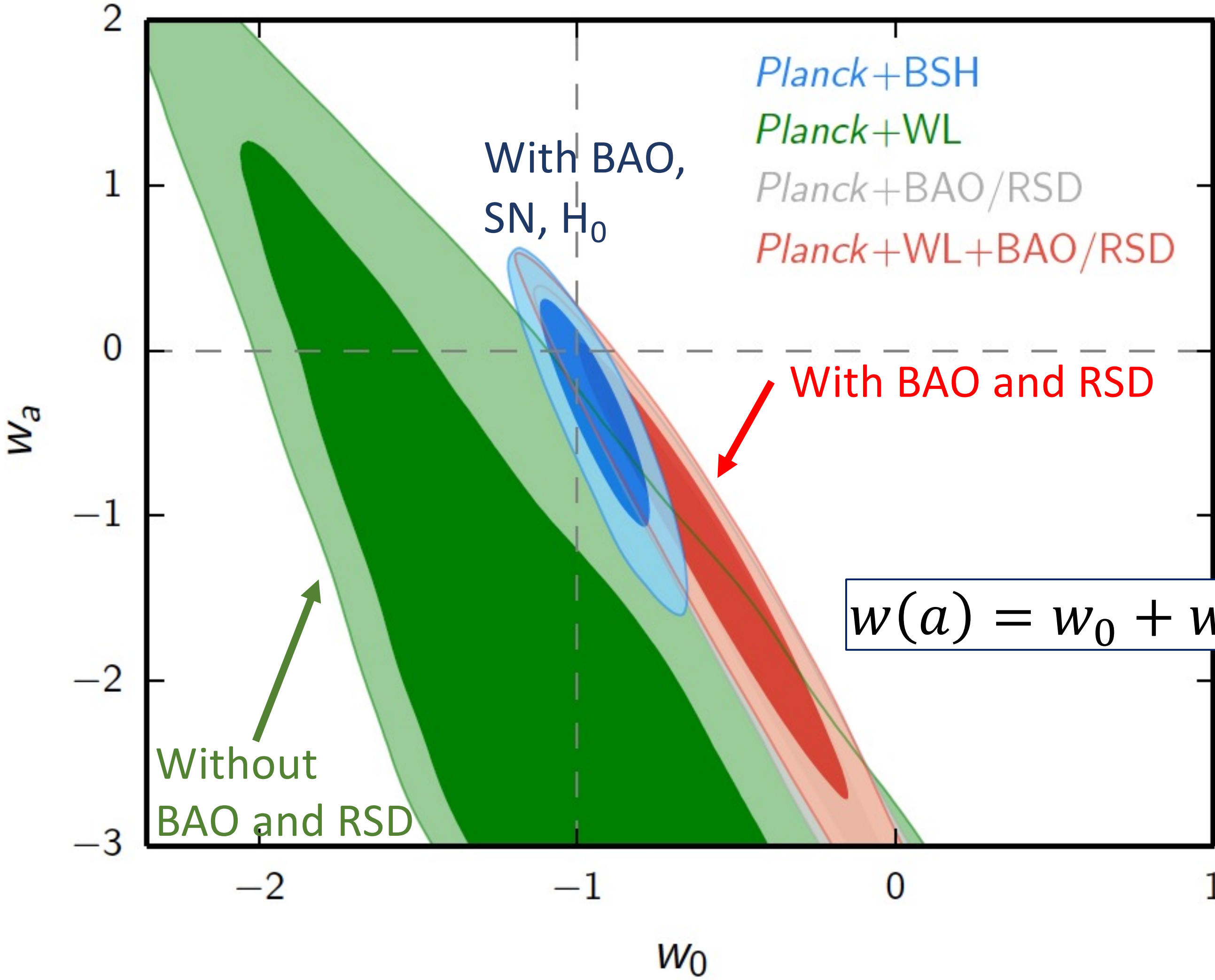
BAO – a powerful tool for precision cosmology





- Adding BAO to CMB measurements shrinks the error contours from red to blue!

BAO – a powerful tool for precision cosmology

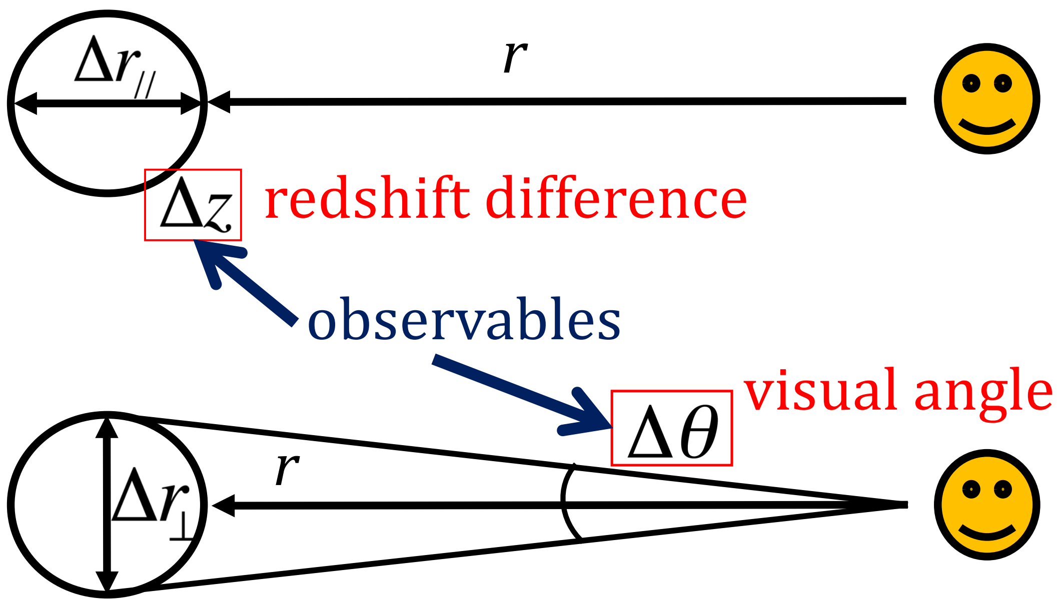


$$w(a) = w_0 + w_a(1 - a)$$

See Francisco Prada's lecture for the latest DESI constraint on dynamical dark energy.

BAO is more than just a Standard ruler

- Assume there is an object whose intrinsic shape is a sphere.



$$\Delta r_{//} = \frac{\Delta z}{H(z)}$$

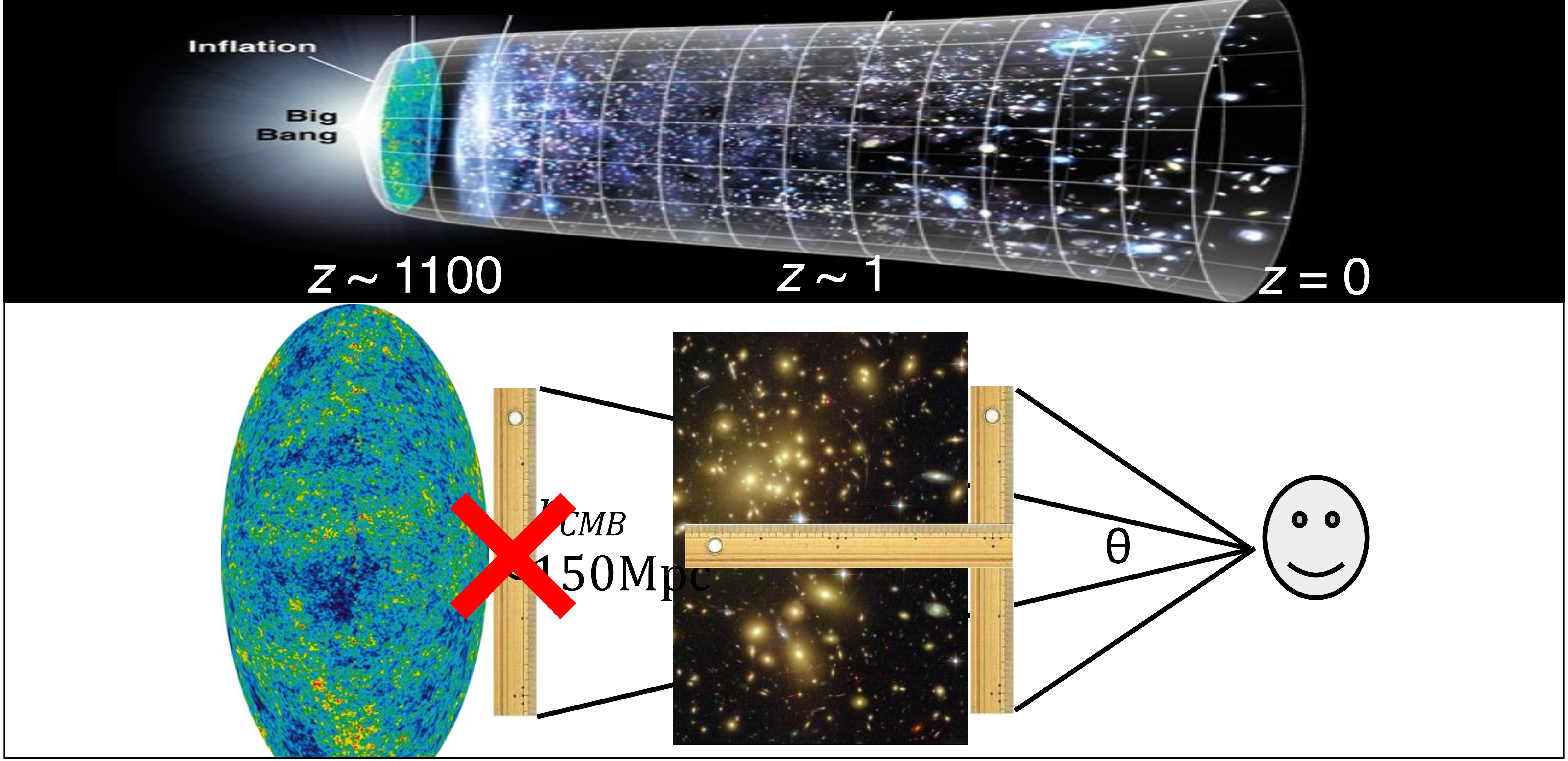
$$\Delta r_{\perp} = (1+z) D_A(z) \Delta\theta$$

$$H(z) = H_0 \sqrt{(1+z)^3 \Omega_{\mathrm{m}} + (1+z)^{3(1+w)} \Omega_{\mathrm{DE}}}$$

$$D_{\mathrm{A}}(z) = (1+z)^{-1} \int_0^z \frac{dz'}{H(z')}$$

- One can determine the geometry so that the object looks like a sphere $\Delta r_{//} = \Delta r_{\perp}$ (Alcock & Paczynski 1979).

BAO is more than a Standard ruler



Anisotropic power spectrum and correlation function

- Linear RSD formula (Kaiser 1987)

$$P_g^S(k,\mu_k;z) = \left(b + f(z)\mu_k^2\right)^2 P_m(k;z)$$

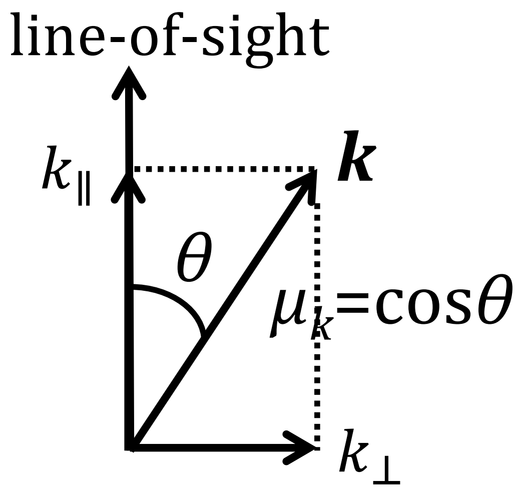
- Including AP effect

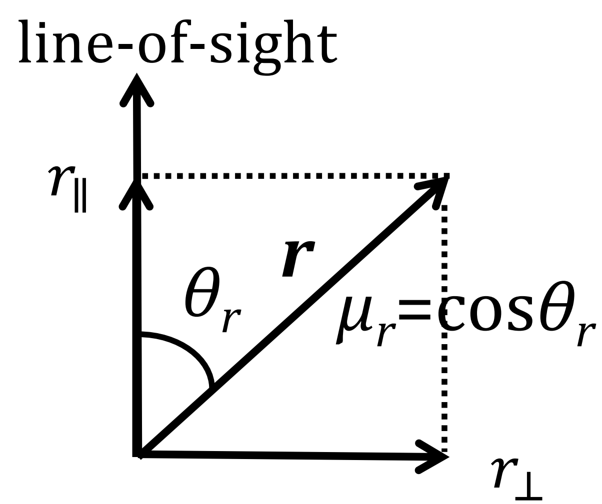
$$P_g^{\rm obs}(k_\perp^{\rm fid},k_\parallel^{\rm fid};z) = \frac{H(z)}{H^{\rm fid}(z)}\left[\frac{D_A^{\rm fid}(z)}{D_A(z)}\right]^2 \left(b + f(z)\mu_k^2\right)^2 P_m(k;z)$$

with $k_\parallel^{\rm fid} = k_\parallel H^{\rm fid}(z)/H(z)$ and $k_\perp^{\rm fid} = k_\perp D_{\rm A}(z)/D_{\rm A}^{\rm fid}(z)$

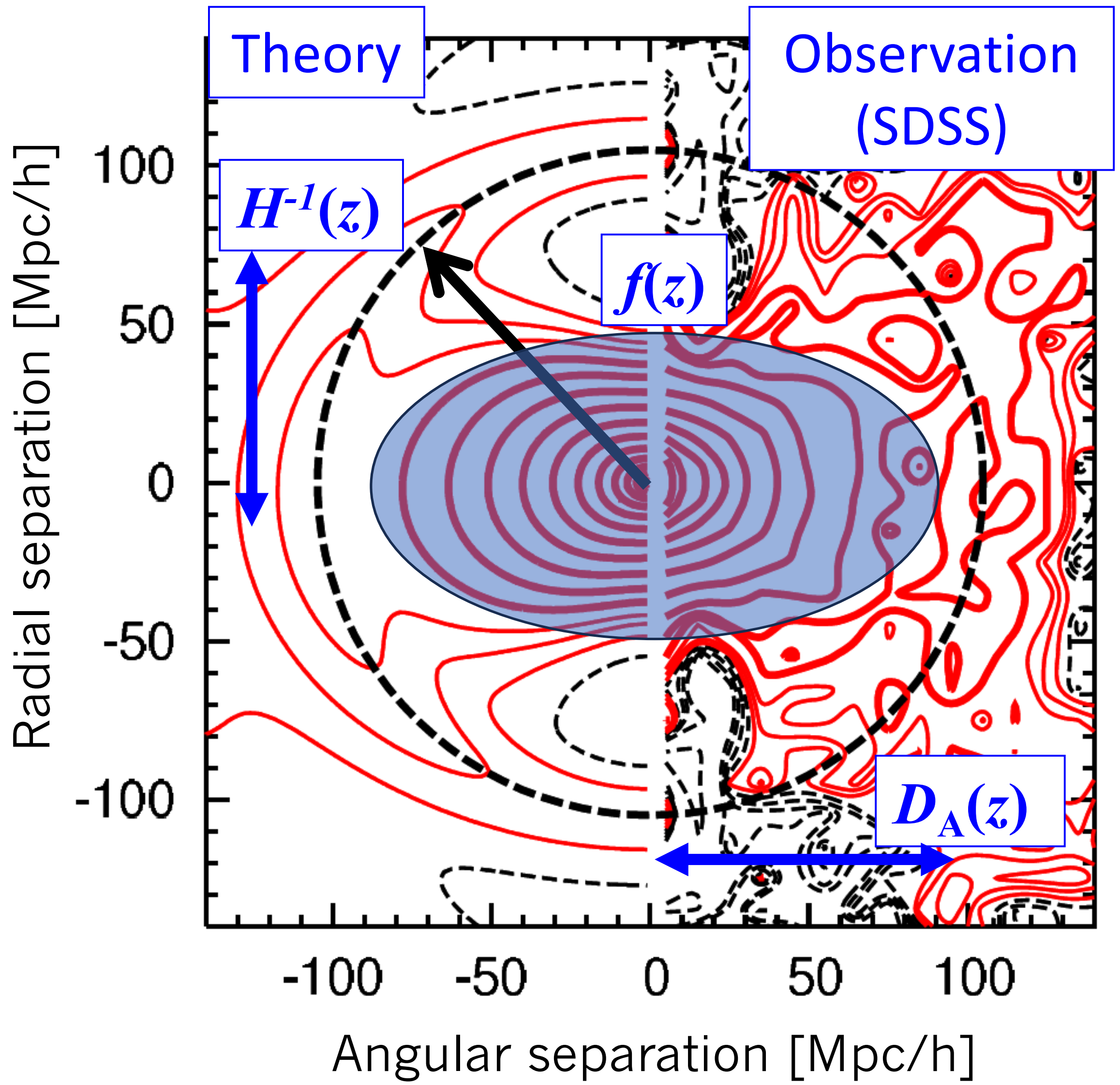
- Correlation function

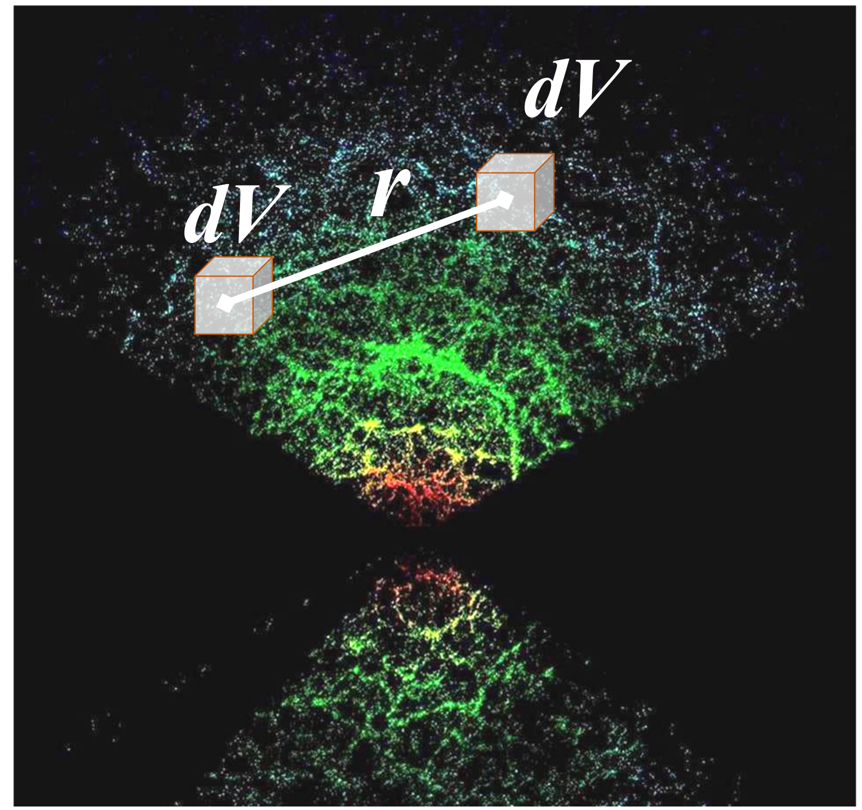
$$\xi_g^{\rm obs}(r_\perp,r_\parallel;z) = \int \frac{d^3k}{(2\pi)^3} P_g^{\rm obs}(k_\perp,k_\parallel;z)e^{i\vec{k}\cdot\vec{r}}$$





First measurement of anisotropy of BAO



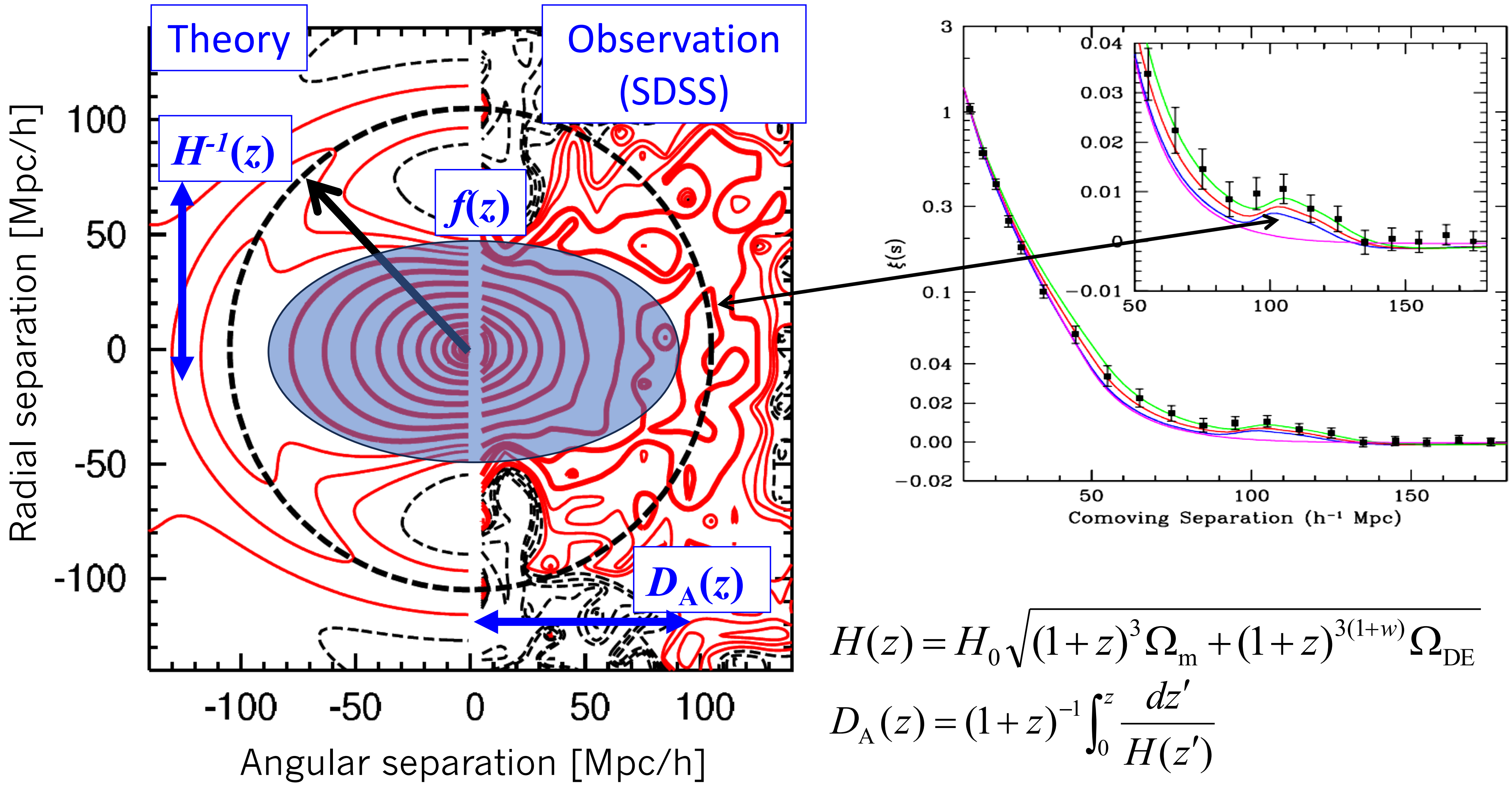


$$H(z) = H_0\sqrt{(1+z)^3\Omega_{\rm m} + (1+z)^{3(1+w)}\Omega_{\rm DE}}$$

$$D_{\rm A}(z) = (1+z)^{-1}\int_0^z \frac{dz'}{H(z')}$$

Okumura, Matsubara, Eisenstein et al (2008)

First measurement of anisotropy of BAO



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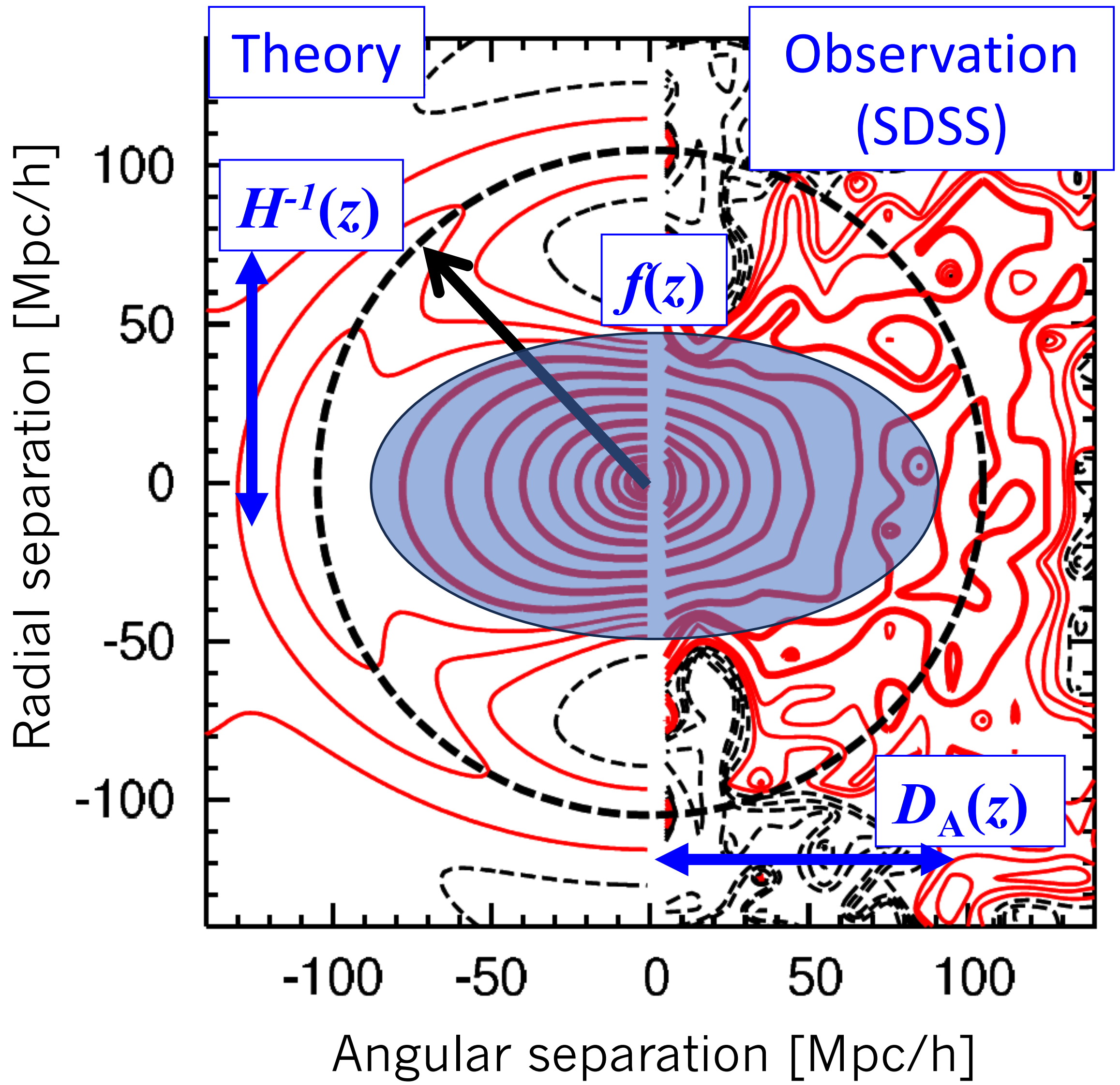
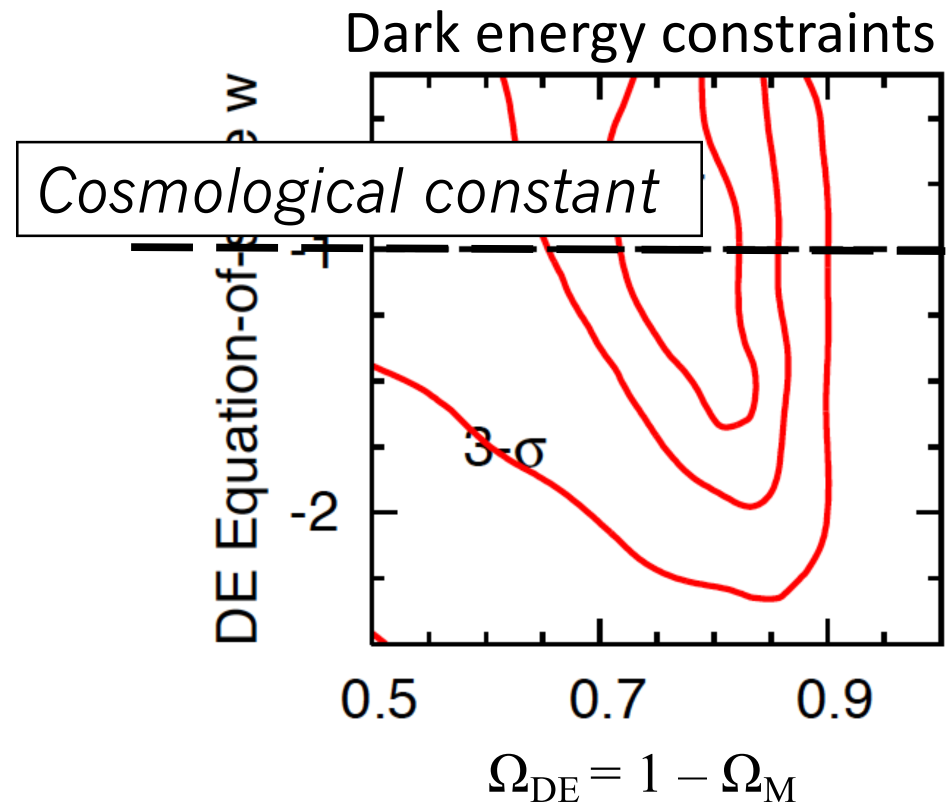
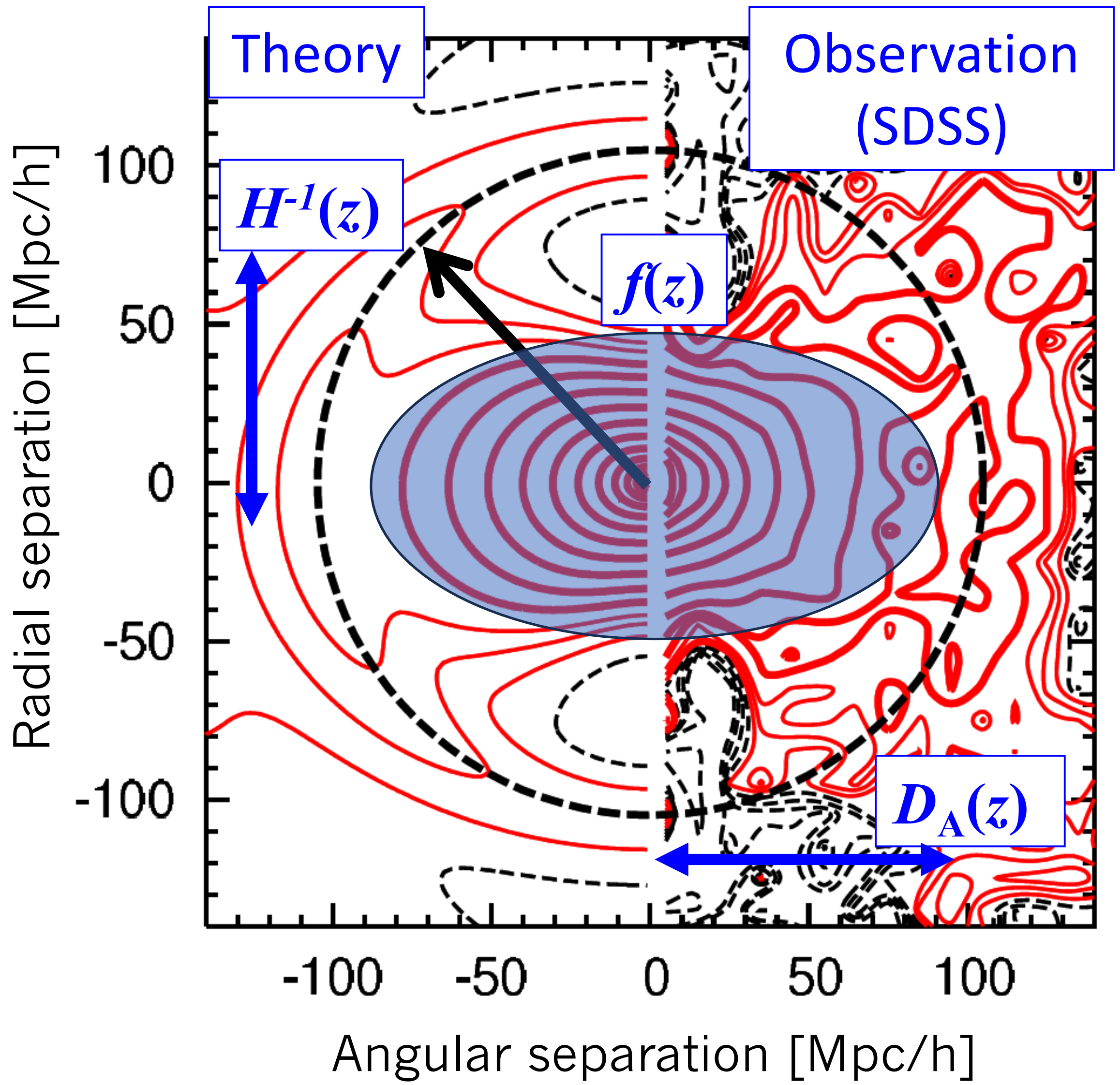


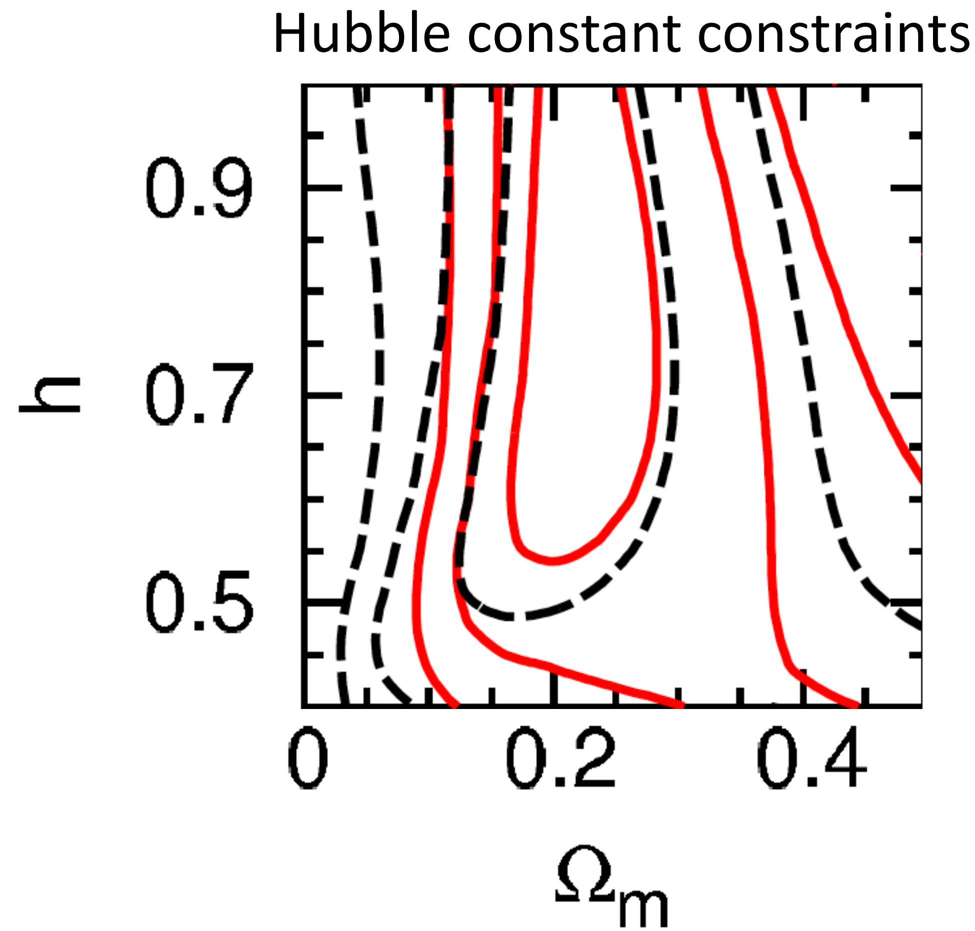


$w_{DE} = -0.93 \pm 0.4$
→consistent with Einstein's cosmological constant $w_{DE} = -1$

Okumura, Matsubara, Eisenstein et al (2008)

First measurement of anisotropy of BAO

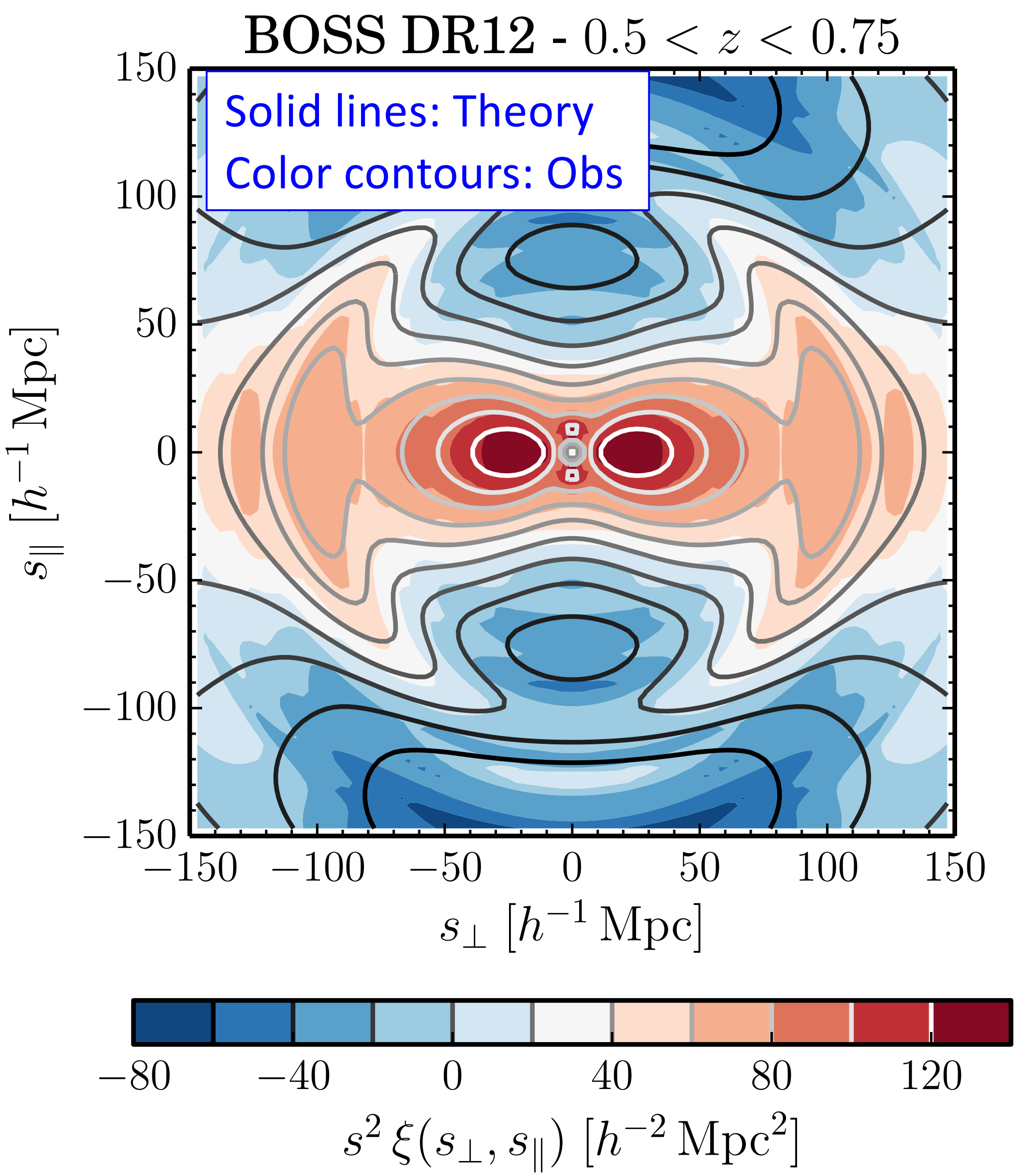




$H_0 = 70.2^{+18.7}_{-11.7}$

Okumura, Matsubara, Eisenstein et al (2008)

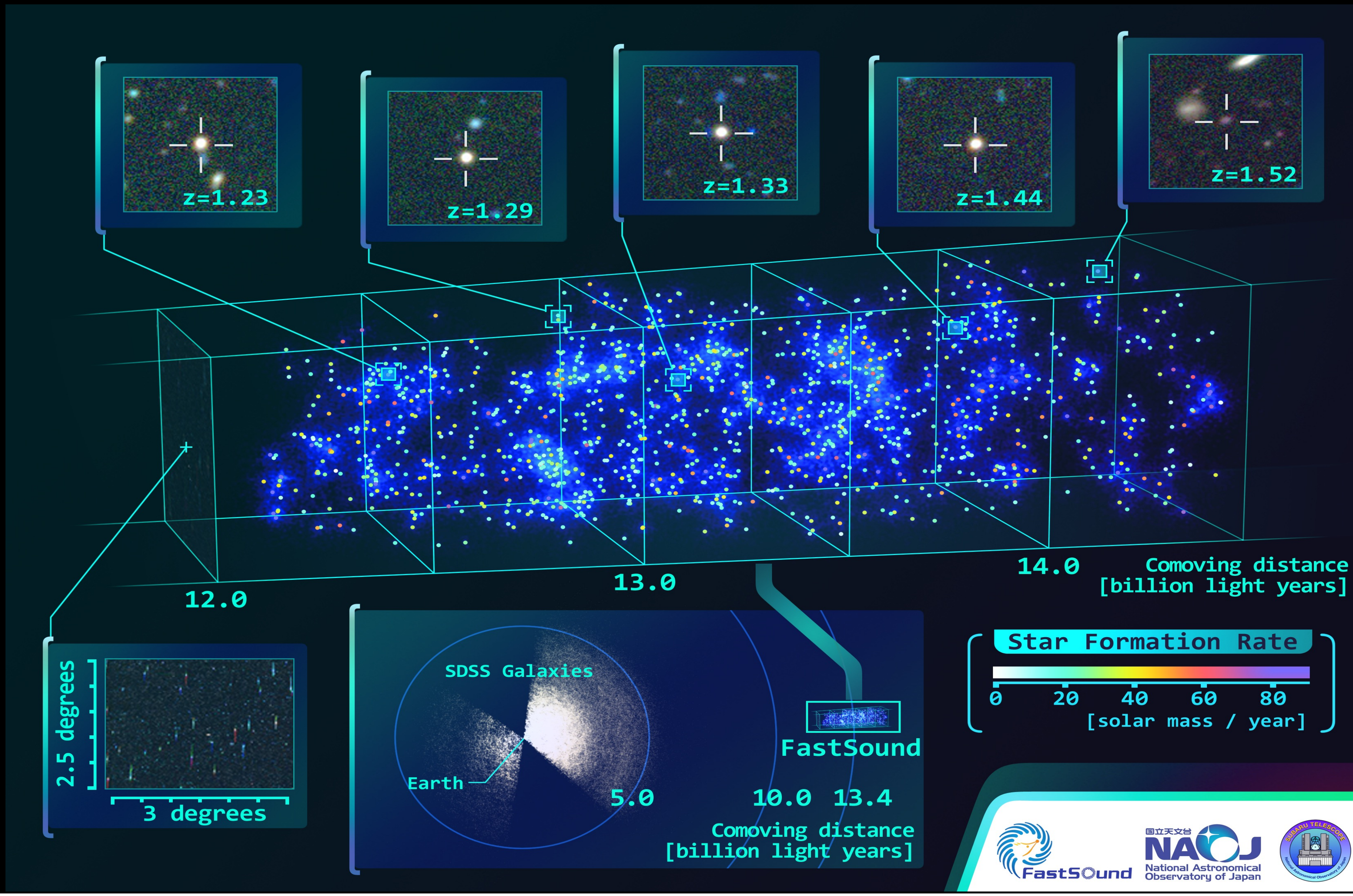
Anisotropic correlation function of BOSS galaxies



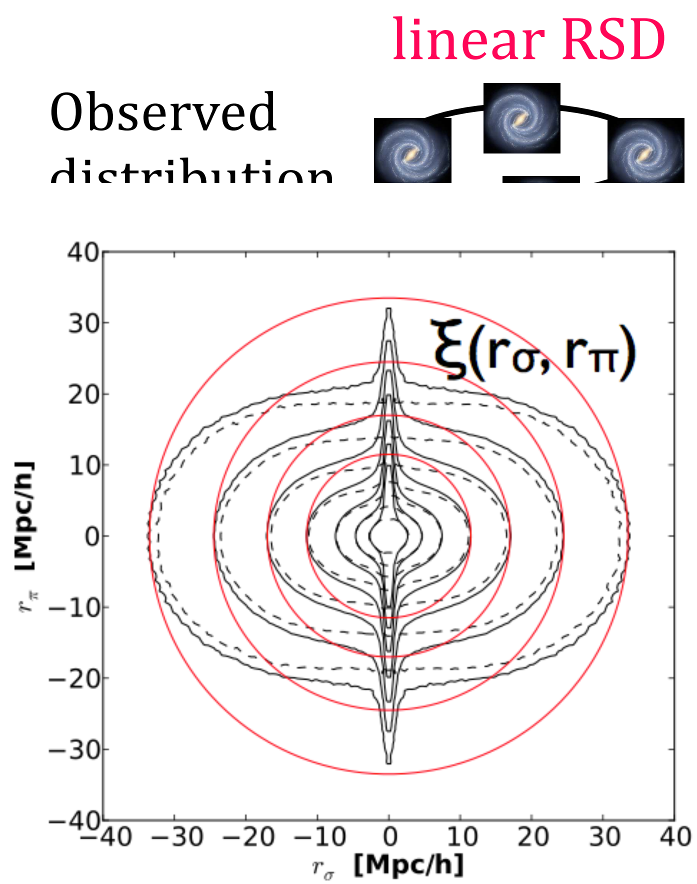
- Correlation function of ~600,000 galaxies in BOSS DR12 sample
- Much clearer anisotropic BAO and RSD signals.

SDSS-III BOSS Collaboration (2017)

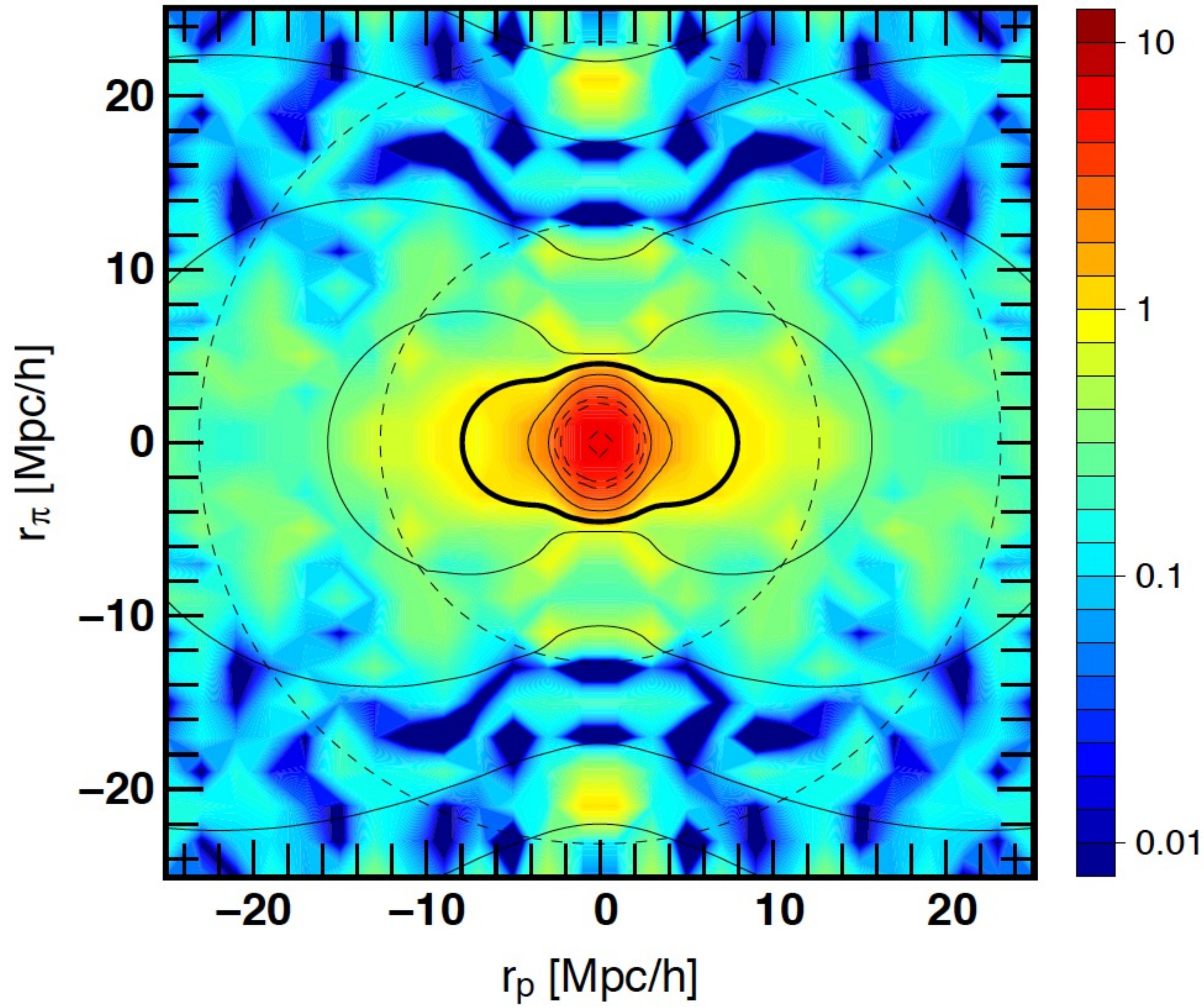
FastSound: first 3d galaxy map at the distant universe



Anisotropic correlation function measured at z~1.4

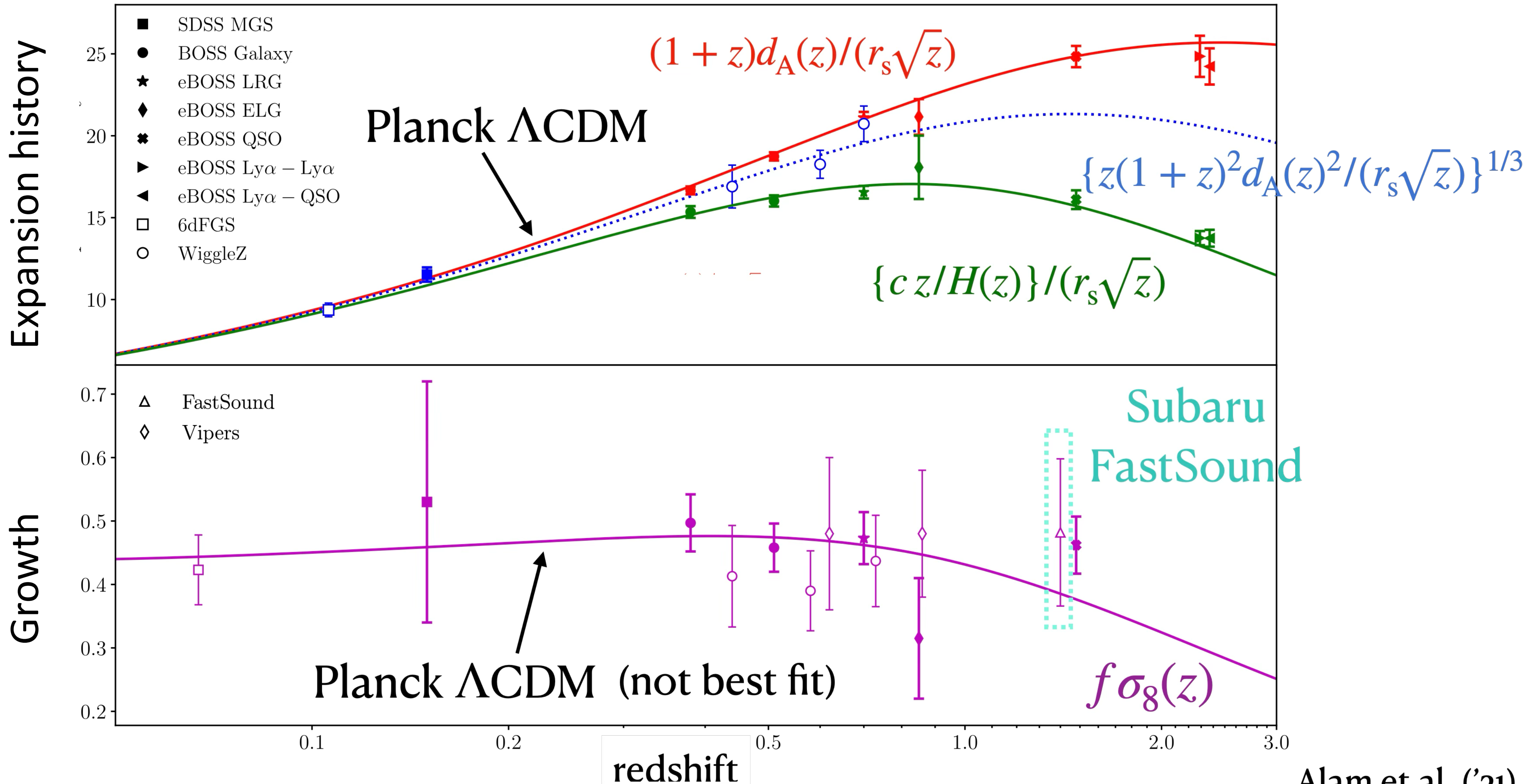


low-z (z~0.5)



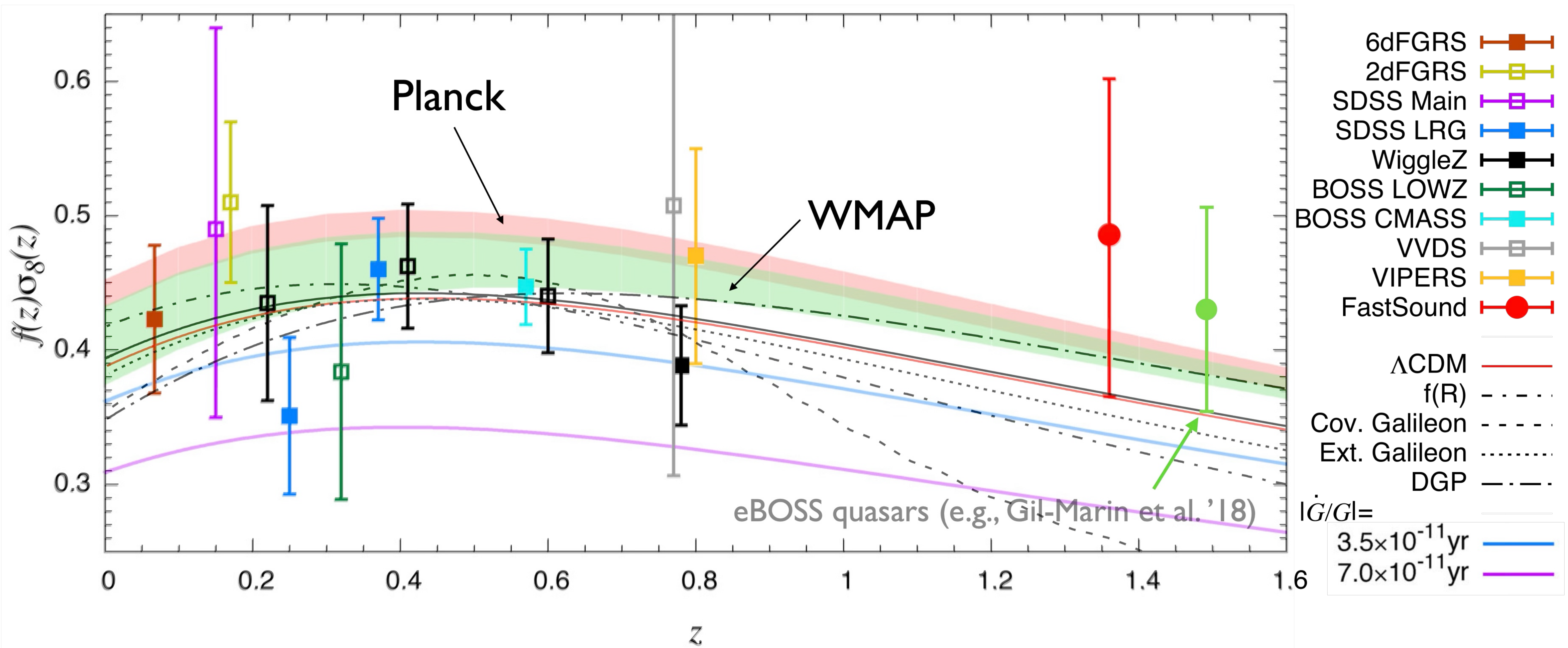
Okumura, Hikage, Totani et al (2016)

First RSD measurement at the distant universe



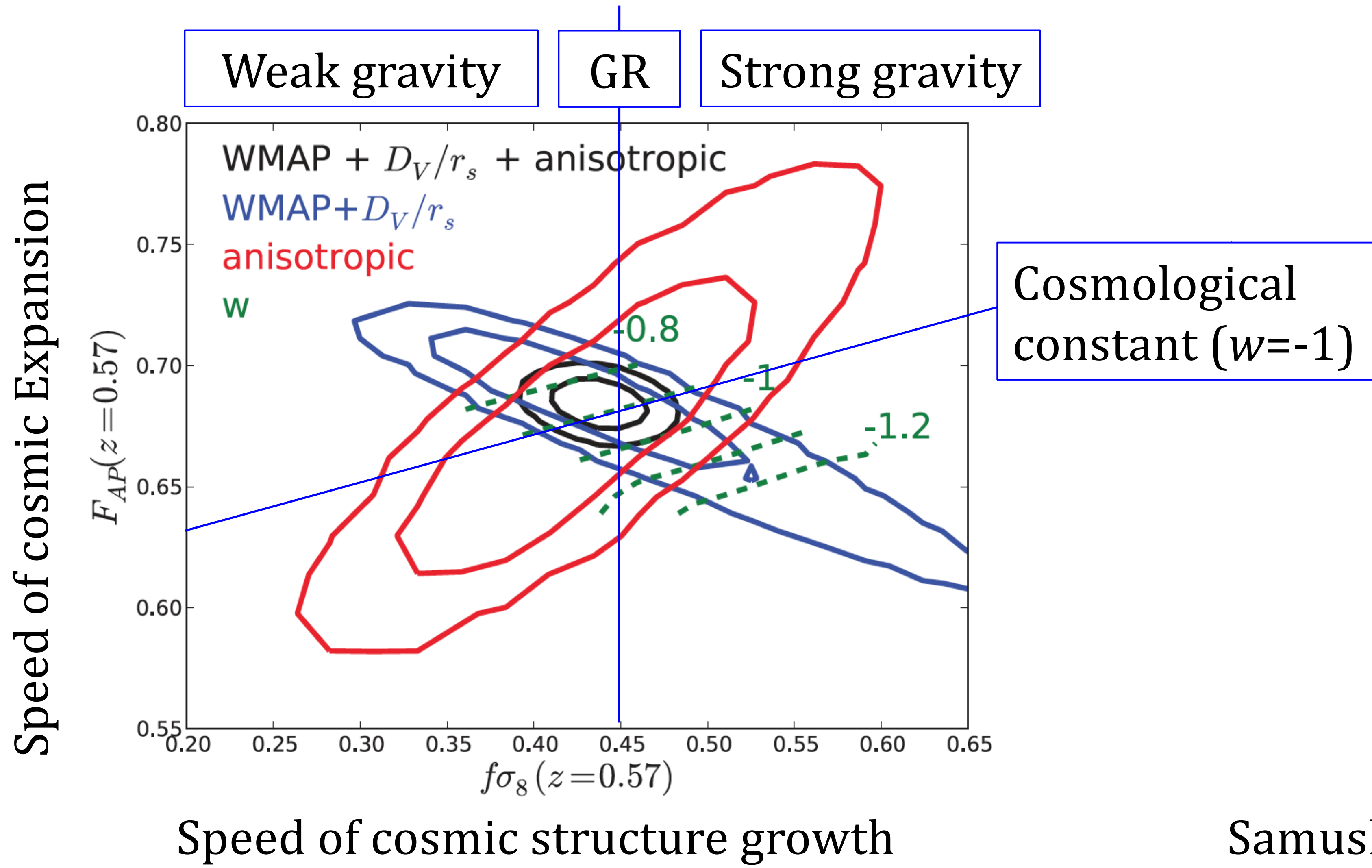
Alam et al. ('21)

Testing gravity from RSD measurements



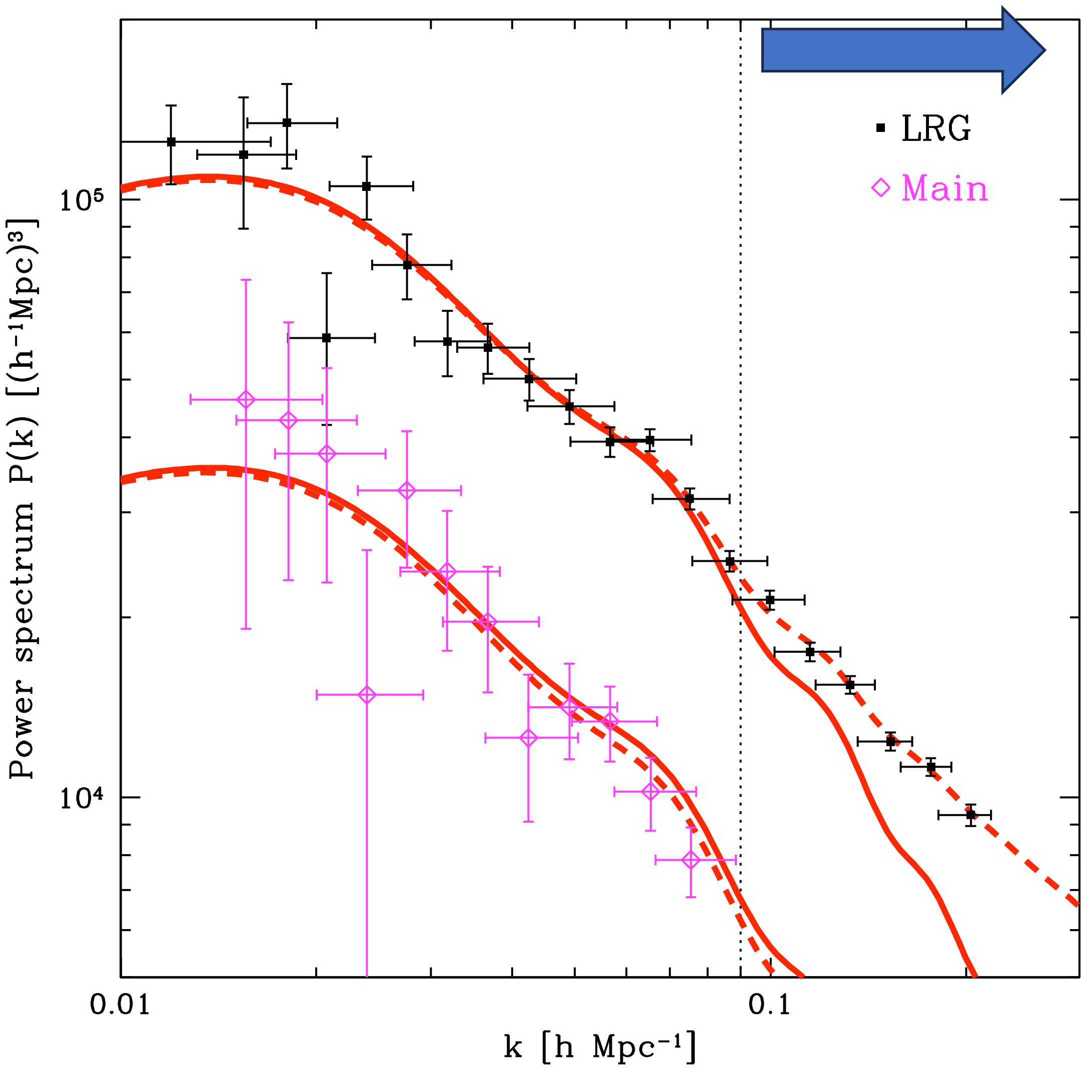
Okumura, Hikage, Totani et al (2016)

Cosmological constant? Dark energy? Or modified gravity?



Samushia+ (2013)

To extract more cosmological information…

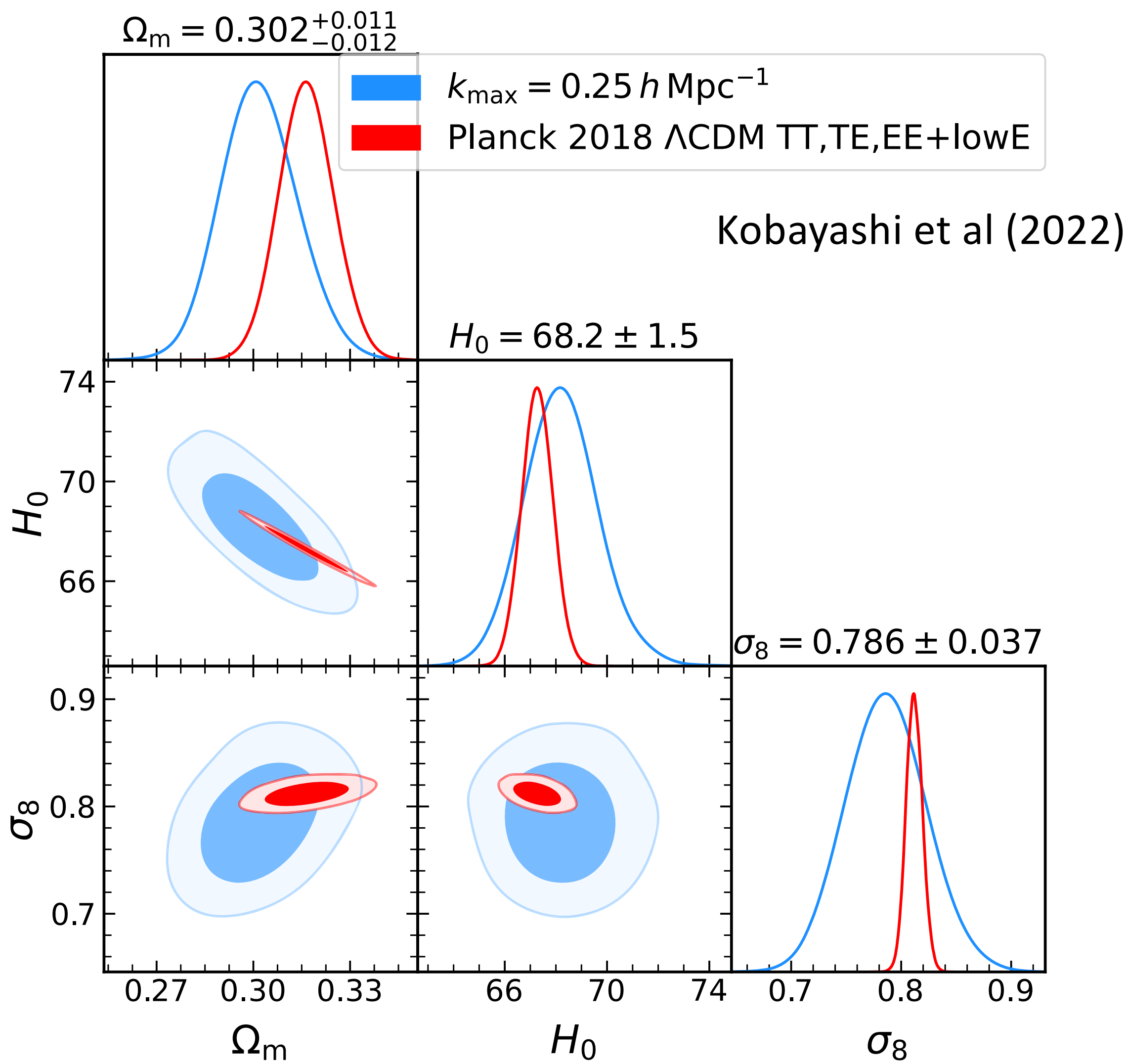


Tegmark et al (2006)

- Cosmological information ∝ (# of modes) ∝ $k_{max}{}^3$
- We need to take into account "nonlinearities" in theoretical models
  - Nonlinear RSD
  - Nonlinear matter evolution
  - Nonlinear bias
- Perturbative approaches
- Simulation-based (emulators)
- Full-shape analysis

$$P_g^{\mathrm{obs}}\left(k_\perp^{\mathrm{fid}}, k_\parallel^{\mathrm{fid}}; z\right) = \frac{H(z)}{H^{\mathrm{fid}}(z)}\left[\frac{D_A^{\mathrm{fid}}(z)}{D_A(z)}\right]^2 \left(b + f(z)\mu_k^2\right)^2 P_m(k; z)$$

Full-shape cosmological constraints using the SDSS-III BOSS data



- They built an emulator from N-body simulations to compute the redshift-space power spectrum of halos.
- A halo model is adopted to model the galaxy-halo connection.
- Very strong constraints are obtained by fully utilizing the full shape information of the matter power spectrum.

See Takahiro Nishimichi's lecture for more details.

Summary

- Galaxy redshift surveys provide one of the best tools to probe the cosmic acceleration.
- Statistical properties of galaxy distributions carry ample cosmological information
  - Redshift-space distortions (RSD): f(z)
  - Baryon acoustic oscillations (BAO): $D_A(z)$ and H(z)
- More information is available from broadband features of the matter power spectrum.
- A proper cosmological interpretation requires a detailed theoretical modeling of nonlinear structure formation.
  - N-body simulations, perturbation theory, etc…

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http://www.asiaa.sinica.edu.tw/~tokumura/

Some recommended textbooks for structure formation and galaxy surveys

- B. Ryden, "*Introduction to Cosmology*" 2nd Ed. (2016)
  - Undergraduate level.
  - No knowledge of general relativity is required.
- S. Dodelson & F. Schmidt, "*Modern Cosmology*" 3rd Ed. (2025)
  - Standard graduate level.
  - A self-consistent textbook that you can derive all the equations by yourself.
- H. Mo, F. van den Bosch, & S. White, "*Galaxy Formation and Evolution*" (2009)
  - Comprehensive description of galaxy formation/evolution and the large-scale structure of the universe. Advanced
- P. J. E. Peebles, "*The Large-Scale Structure of the Universe*"(1980)
  - Very advanced. Some parts are outdated.
  - Describes all the essential statistical tools for galaxy survey analysis.